FACTORS ON DEMAND

Optimized Flexible Factors for Risk Estimation and Attribution

Attilio Meucci

http://ssrn.com/abstract=1565134
EXECUTIVE SUMMARY

TRADITIONAL MULTI-PURPOSE FACTOR MODELS

FACTORS ON DEMAND – THEORY

FACTORS ON DEMAND – APPLICATIONS

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### Executive Summary

**Risk Estimation vs. Risk Attribution**

<table>
<thead>
<tr>
<th>Risk Estimation</th>
<th>Risk Attribution</th>
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</thead>
<tbody>
<tr>
<td>• Identify Risk Factors to impose structure on estimate of large multivariate market distribution</td>
<td>• Define Attribution Factors</td>
</tr>
<tr>
<td>• Compute overall portfolio risk (Standard Deviation and Tail Risk) from market distribution</td>
<td>• Allocate overall portfolio risk obtained from Risk Estimation to Attribution Factors</td>
</tr>
<tr>
<td>• <strong>Goal</strong>: maximize predictive power</td>
<td>• <strong>Goal</strong>: maximize interpretability and practicality for hedging/trading</td>
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**Risk Estimation**
- Identify Risk Factors \( F_k \) to impose structure on estimate of large multivariate market distribution
- Compute overall portfolio risk (Standard Deviation and Tail Risk) from market distribution
- **Goal**: maximize predictive power

**Risk Attribution**
- Define Attribution Factors \( F_k \)
- Allocate overall portfolio risk obtained from Risk Estimation to Attribution Factors
- **Goal**: maximize interpretability and practicality for hedging/trading

**Traditional Factor Models**: same or similar factors for Risk Estimation and Attribution

- Suboptimal choice of “systematic” factors
  - Suboptimal statistical properties for risk estimation
  - Risk attribution factors are not most practical for hedging/interpretation
  - Not portfolio-specific estimation/attribution
- Inflexible choice of loadings (“betas”)
  - Rigid bottom-up aggregation (beta of portfolio is sum of beta of securities)
  - Rigid maximization target (R-square)
  - Rigid unconstrained maximization (CAPM beta)
- Incorrect modeling of non-linear products/derivatives
**Risk Estimation**
- Identify Risk Factors $F_k$ to impose structure on estimate of large multivariate market distribution
- Compute overall portfolio risk (Standard Deviation and Tail Risk) from market distribution
- **Goal**: maximize predictive power

**Risk Attribution**
- Define Attribution Factors $Z_k$
- Allocate overall portfolio risk obtained from Risk Estimation to Attribution Factors
- **Goal**: maximize interpretability and practicality for hedging/trading

**Factors On Demand**: different factors for Risk Estimation and Risk Attribution

- Flexible choice of factors: “dominant”, instead of “systematic”
  - Ideal statistical properties for risk estimation
  - Ideal hedging/interpretation properties for risk attribution
  - Portfolio-specific estimation/attribution
- Flexible choice of loadings (“betas”)
  - Flexible top-down aggregation
  - Flexible maximization target (R-square, CVaR, etc.)
  - Flexible constrained maximization (best pool, long-only, etc.)
- Consistent across non-linear products/derivatives (full conditional distribution of $Z_k$)
EXECUTIVE SUMMARY

TRADITIONAL MULTI-PURPOSE FACTOR MODELS

FACTORS ON DEMAND – THEORY

FACTORS ON DEMAND – APPLICATIONS

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Risk Estimation

1. Stocks return estimation

\[ R_n = \sum_k b_{n,k} F_k + U_n \]

- \( R_n \) = security return
- \( b_{n,k} \) = loading
- \( F_k \) = systematic factor
- \( U_n \) = idiosyncratic shock

Risk Estimation Rationales
- Estimate the joint distribution of security returns, imposing structure with factor model

Traditional Risk Estimation Techniques
- Regression analysis
Risk Estimation

1. Stocks return estimation

\[ R_n = \sum_k b_{n,k} F_k + U_n \]

- \( R_n \) = security return
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3. Aggregation

\[ R_w = \sum_n w_n R_n \]

4. Portfolio risk estimation

- Standard deviation
  \[ \text{Stdev} \{ R_w \} = w' [b \Sigma_F b' + \text{diag}(\sigma_U^2)] w \]
- Value at Risk
  \[ \text{VaR} \{ R_w \} \text{ Normal assumption} \]

Risk Estimation Rationales
- Estimate the joint distribution of security returns, imposing structure with factor model
- Use the portfolio positions \( w \) to determine aggregated portfolio return distribution
- Define and compute risk: standard deviation, Value at Risk (tail risk), etc.

Traditional Risk Estimation Techniques
- Regression analysis
- Dimension reduction
- Parametric assumptions
Risk Estimation

1. Stocks return estimation

\[
R_n = \sum_k b_{n,k} F_k + U_n
\]

- \( R_n \) = security return
- \( b_{n,k} \) = loading
- \( F_k \) = systematic factor
- \( U_n \) = idiosyncratic shock

\[
R_n = g(X_1, \ldots, X_S)
\]

Traditional modeling of non-linear securities

- For non-equity securities such as bonds and derivatives, the returns \( R \) are not “invariants”, i.e. they do not behave identically and independently across time

Example: bond

\[
R = \frac{P(X_1, X_2)}{P_0} - 1
\]

- \( P \) : discount formula
- \( X_1 \) : govt curve changes
- \( X_2 \) : spread changes

Example: option

\[
R = \frac{BS(X_1, X_2)}{P_0} - 1
\]

- \( BS \) : Black-Scholes formula
- \( X_1 \) : log-return of underlying
- \( X_2 \) : log-return of implied vol.
**Risk Estimation**

1. **Stocks return estimation**

\[
R_n = \sum_k b_{n,k} F_k + U_n
\]

- \(R_n\) = security return
- \(b_{n,k}\) = loading
- \(F_k\) = systematic factor
- \(U_n\) = idiosyncratic shock

2. **Risks drivers estimation**

\[
X_s = \sum_k b_{s,k} F_k + U_s
\]

- \(X_s\) = risk driver
- \(b_{s,k}\) = loading
- \(F_k\) = systematic factor
- \(U_s\) = idiosyncratic shock

3. **Pricing**

\[
R_n = g(X_1, \ldots, X_S)
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---

**Traditional modeling of non-linear securities**

- For non-equity securities such as bonds and derivatives, the returns \(R\) are not “invariants”, i.e. they do not behave identically and independently across time.
- Therefore, estimation cannot be performed on returns, but rather on risk drivers \(X\), which are “invariants”

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### Traditional modeling of non-linear securities

- For non-equity securities such as bonds and derivatives, the returns $R$ are not “invariants”, i.e. they do not behave identically and independently across time.
- Therefore, estimation cannot be performed on returns, but rather on risk drivers $X$, which are “invariants”.
- Then, risk drivers $X$ are transformed into returns $R$ by “delta” or “duration” coefficients $\delta$.

**Example: bond**

\[
R \approx \delta_1 X_1 + \delta_2 X_2
\]

- $\delta_1$: curve duration
- $\delta_2$: spread duration
- $X_1$: govt curve changes
- $X_2$: spread changes

**Example: option**

\[
R \approx \delta_1 X_1 + \delta_2 X_2
\]

- $\delta_1$: delta
- $\delta_2$: vega
- $X_1$: log-return of underlying
- $X_2$: log-return of implied vol.
Risk Estimation

1. Risk drivers estimation
\[ X_s = \sum_k b_{s,k} F_k + U_s \]
\[ \begin{align*} X_s &= \text{risk driver} \\ b_{s,k} &= \text{loading} \\ F_k &= \text{systematic factor} \\ U_n &= \text{idiosyncratic shock} \]

2. Pricing
\[ R_n \approx \sum_s \delta_{n,s} X_s \]

3. Aggregation
\[ R_w = \sum_n w_n R_n \]

4. Portfolio risk estimation
\[ \text{Sdev} \{R_w\} = w' \delta \left[ b \Sigma_F b' + \text{diag} \left( \sigma_U^2 \right) \right] \delta' w \]
\[ \text{VaR} \{R_w\} = \text{Normal assumption} \]

Traditional modeling of non-linear securities
- For non-equity securities such as bonds and derivatives, the returns \( R \) are not “invariants”, i.e. they do not behave identically and independently across time
- Therefore, estimation cannot be performed on returns, but rather on risk drivers \( X \), which are “invariants”
- Then, risk drivers \( X \) are transformed into returns \( R \) by “delta” or “duration” coefficients \( \delta \)
- The risk computations follow
1. Risk drivers estimation
\[ X_s = \sum_k b_{s,k} F_k + U_s \]
- \( X_s \): risk driver
- \( b_{n,k} \): loading
- \( F_k \): systematic factor
- \( U_n \): idiosyncratic shock

2. Pricing
\[ R_n \approx \sum_s \delta_{n,s} X_s \]

3. Aggregation
\[ R_w = \sum_n w_n R_n \]

4. Portfolio risk estimation
\[
\begin{align*}
\text{Sdev} \{ R_w \} &= w' \delta \left[ b \Sigma_F b' + \text{diag} \left( \sigma_U^2 \right) \right] \delta' w \\
\text{VaR} \{ R_w \} &= \text{Normal assumption}
\end{align*}
\]

5. Attribution factors
\[ F_k \]

6. Security-level attribution
\[ R_n = \sum_k b_{n,k} F_k + U_n \]
\[ b_{n,k} = \sum_s \delta_{n,s} b_{s,k} \]

Risk Attribution Rationales
- After obtaining aggregate portfolio risk (Sdev, VaR, CVaR, etc.), attribute it to individual factors
- Purpose: see how factors contributed to portfolio risk and make hedging decision

Traditional Risk Attribution Techniques
- Use same factors for attribution as for estimation
- Perform linear operations to define security-level risk attribution
**Risk Attribution**

1. Risk drivers estimation
   \[ X_s = \sum_k b_{s,k} F_k + U_s \]
   - \( X_s \) = risk driver
   - \( b_{n,k} \) = loading
   - \( F_k \) = systematic factor
   - \( U_n \) = idiosyncratic shock

2. Pricing
   \[ R_n \approx \sum_s \delta_{n,s} X_s \]

3. Aggregation
   \[ R_w = \sum_n w_n R_n \]

4. Portfolio risk estimation
   \[ S_{dev} \{ R_w \} = w' \delta \left[ \sum F b' + \text{diag} \left( \sigma^2_U \right) \right] \delta' w \]
   \[ V_{aR} \{ R_w \} = \text{Normal assumption} \]

**Traditional Multi-Purpose Factor Models**

5. Attribution factors
   \[ F_k \]

6. Security-level attribution
   \[ R_n = \sum_k b_{n,k} F_k + U_n \]
   \[ b_{n,k} = \sum_s \delta_{n,s} s_{s,k} \]

7. Portfolio risk attribution: bottom up
   \[ R_w = \sum_k b_{w,k} F_k + U_w \]
   \[ b_{w,k} = \sum_n w_n b_{n,k} \]

**Risk Attribution Rationales**
- After obtaining aggregate portfolio risk (Sdev, VaR, CVaR, etc.), attribute it to individual factors
- Purpose: see how factors contributed to portfolio risk and make hedging decision

**Traditional Risk Attribution Techniques**
- Use same factors for attribution as for estimation
- Perform linear operations to define security-level risk attribution
- Perform bottom-up aggregation for portfolio-level risk attribution
### Risk Estimation

1. **Risk drivers estimation**
   \[ X_s = \sum_k \beta_{s,k} F_k + U_s \]
   - \( X_s \): risk driver
   - \( \beta_{s,k} \): loading
   - \( F_k \): systematic factor
   - \( U_s \): idiosyncratic shock

2. **Pricing**
   \[ R_n \approx \sum_s \delta_{n,s} X_s \]

3. **Aggregation**
   \[ R_w = \sum_n w_n R_n \]

4. **Portfolio risk estimation**
   \[ \text{Sdev} \{ R_w \} = w' \delta \left[ b \Sigma_F b' + \text{diag} \left( \sigma_U^2 \right) \right] \delta' w \]
   \[ \text{Var} \{ R_w \} = \text{Normal assumption} \]

### Risk Attribution

5. **Attribution factors**
   \[ F_k \]

6. **Security-level attribution**
   \[ R_n = \sum_k \beta_{n,k} F_k + U_n \]
   - \( \beta_{n,k} = \sum_s \delta_{n,s} \beta_{s,k} \)

7. **Portfolio risk attribution: bottom up**
   \[ R_w = \sum_k \beta_{w,k} F_k + U_w \]
   - \( \beta_{w,k} = \frac{\text{Cov} \{ R_w, F \}}{\text{Cov} \{ F \}} \)
   - \( \beta_{w,k} = \arg\min_{\beta} \text{Var} \{ R_w - \sum_k \beta_k F_k \} \)

### Pitfalls

- Same factors used for both estimation and attribution: choice neither optimizes the estimation power nor the interpretability or practicality for hedging
- As an estimation model, \( b \) and \( F \) maximize r-square
- As an attribution model, \( b \) and \( F \) maximize r-square (CAPM)
- “delta” assumption can be inappropriate
- Bottom-up aggregation not flexible: small exposures better in residual
**Risk Estimation**
1. Risk drivers estimation
   \[ X_s = \sum_k b_{s,k} F_k + U_s \]
   - \( X_s \): risk driver
   - \( b_{s,k} \): loading
   - \( F_k \): systematic factor
   - \( U_s \): idiosyncratic shock

2. Pricing
   \[ R_n \approx \sum_s \delta_{n,s} X_s \]

3. Aggregation
   \[ R_w = \sum_n w_n R_n \]

4. Portfolio risk estimation
   \[ \text{Sdev}\{R_w\} = w' \delta [b \Sigma_F b' + \text{diag}(\Sigma_U)] \delta' w \]
   \[ \text{VaR}\{R_w\} = \text{Normal assumption} \]

**Risk Attribution**
5. Enhanced attribution factors
   \[ \tilde{F}_j = \sum_k a_{j,k} F_k \quad F_k = \sum_j a_{k,j}^{-1} \tilde{F}_j \]

6. Security-level attribution
   \[ R_n = \sum_j \tilde{b}_{n,j} \tilde{F}_j + U_n \]
   \[ \tilde{b}_{n,j} = \sum_k \delta_{n,s} b_{s,k} a_{k,j}^{-1} \]

7. Portfolio risk attribution: bottom up
   \[ R_w = \sum_j \tilde{b}_{w,j} \tilde{F}_j + U_w \]
   \[ \tilde{b}_{w,j} = \sum_n w_n \tilde{b}_{n,j} \]

**Pitfalls**
- Similar factors used for both estimation and attribution: choice neither optimizes the estimation power nor the interpretability or practicality for hedging
- Factors restricted by the “systematic + idiosyncratic” assumption
- As an estimation model, \( b \) and \( F \) maximize r-square
- As an attribution model, \( b \) and \( F \) maximize r-square (CAPM)
- “delta” assumption can be inappropriate
- Bottom-up aggregation not flexible: small exposures better in residual
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FACTORS ON DEMAND – APPLICATIONS

REFERENCES
### Risk Estimation

1. Risk drivers estimation

\[
\sum_{k} F_k
\]

\[F_k\] = dominant factor

---

**E.g. PCA facts**

\[
\begin{bmatrix}
F_1 & F_K
\end{bmatrix}
\]

1 joint scenario
Risk Estimation

1. Risk drivers estimation

\[ X_s = \sum_k b_{s,k} F_k + U_s \]

- \( X_s \): risk driver
- \( b_{n,k} \): loading
- \( F_k \): dominant factor
- \( U_n \): residual

E.g. PCA facts, PCA res, stocks log.rets
Risk Estimation

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\[
\begin{align*}
X_s & = \text{risk driver} \\
b_{n,k} & = \text{loading} \\
F_k & = \text{dominant factor} \\
U_n & = \text{residual}
\end{align*}
\]

2. Pricing
\[ R_n = g_n (X_1, \ldots, X_S) \]
Risk Estimation

1. Risk drivers estimation

\[ X_s = \sum_k b_{s,k} F_k + U_s \]

- \( X_s \) = risk driver
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\( X_s = \) risk driver
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2. Pricing
\[ R_n = g_n (X_1, \ldots, X_S) \]

3. Aggregation
\[ R_w = \sum_n w_n R_n \]

E.g. PCA facts  PCA res  stocks log.rets  stocks lin. rets  port ret

1 joint scenario
Risk Estimation

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\[ X_s = \sum_k b_{s,k} F_k + U_s \]
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2. Pricing
\[ R_n = g_n (X_1, \ldots, X_S) \]

3. Aggregation
\[ R_w = \sum_n w_n R_n \]

4. Portfolio risk estimation
\[ \text{Sdev} \{ R_w \} = \text{exact} \]
\[ \text{VaR} \{ R_w \} = \text{exact} \]

E.g. PCA facts | PCA res | stocks log.rets | stocks lin. rets | port ret
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</tr>
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<tbody>
<tr>
<td>( F_1 )</td>
<td>( F_K )</td>
<td>( U_1 )</td>
<td>( U_S )</td>
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1 joint scenario
Factors On Demand
Attilio Meucci

1. Risk drivers estimation
\[ X_s = \sum_k b_{s,k} F_k + U_s \]
\[ \begin{align*}
X_s &= \text{risk driver} \\
F_k &= \text{dominant factor} \\
U_s &= \text{residual} \\
b_{s,k} &= \text{loading} \\
\end{align*} \]

2. Pricing
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\[ \text{Sdev} \{ R_w \} = \text{exact} \]
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5. Attribution factors
\[ Z_k | X_1, \ldots, X_S \]

Diagram:
- E.g.: PCA facts, PCA res, stocks log.rets, stocks lin. rets, port ret, attribution, GICS sectors
- Conditional scenarios given \( X \)
1. Risk drivers estimation

\[ X_s = \sum_k b_{s,k} F_k + U_s \]

- \( X_s \) = risk driver
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4. Portfolio risk estimation

- \( Sdev \{ R_w \} \) = exact
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5. Attribution factors

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E.g. PCA facts, PCA res, stocks log.rets, stocks lin. rets, port ret, attribution, GICS sectors

Conditional scenarios given \( X \)
# FACTORS ON DEMAND

**Attilio Meucci**

## Risk Estimation

1. **Risk drivers estimation**
   \[
   X_s = \sum_k b_{s,k} F_k + U_s
   \]
   \(X_s\) = risk driver  
   \(b_{s,k}\) = loading  
   \(F_k\) = dominant factor  
   \(U_s\) = residual

2. **Pricing**
   \[
   R_n = g_n (X_1, \ldots, X_S)
   \]

3. **Aggregation**
   \[
   R_w = \sum_n w_n R_n
   \]

4. **Portfolio risk estimation**
   \[
   \text{Sdev} \{R_w\} = \text{exact} \\
   V\alpha R \{R_w\} = \text{exact}
   \]

## Risk Attribution

5. **Attribution factors**
   \[
   Z_k | X_1, \ldots, X_S
   \]

---

---

**E.g.**

<table>
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<tr>
<th>PCA facts</th>
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<td>(X_S)</td>
<td>(Z_1)</td>
</tr>
<tr>
<td>(b_{s,k})</td>
<td></td>
<td></td>
<td></td>
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1 joint scenario

conditional scenarios given \(X\)
### Risk Estimation

1. **Risk drivers estimation**
   \[ X_s = \sum_k b_{s,k} F_k + U_s \]
   - \( X_s \): risk driver
   - \( b_{n,k} \): loading
   - \( F_k \): dominant factor
   - \( U_n \): residual

2. **Pricing**
   \( R_n = g_n (X_1, \ldots, X_S) \)

3. **Aggregation**
   \( R_w = \sum_n w_n R_n \)

4. **Portfolio risk estimation**
   \[ Sdev \{ R_w \} = \text{exact} \]
   \[ Var \{ R_w \} = \text{exact} \]

### Risk Attribution

5. **Attribution factors**
   \[ Z_k | X_1, \ldots, X_S \]

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**FACTORS ON DEMAND**

Attilio Meucci

**FOD – Theory**

**Risk Estimation**

1. Risk drivers estimation
   \[ X_s = \sum_k b_{s,k} F_k + U_s \]
   - \( X_s \): risk driver
   - \( b_{n,k} \): loading
   - \( F_k \): dominant factor
   - \( U_n \): residual

2. Pricing
   \[ R_n = g_n (X_1, \ldots, X_S) \]

3. Aggregation
   \[ R_w = \sum_n w_n R_n \]

4. Portfolio risk estimation
   \[ Sdev \{ R_w \} = \text{exact} \]
   \[ Var \{ R_w \} = \text{exact} \]

---

**Risk Attribution**

5. Attribution factors
   \[ Z_k | X_1, \ldots, X_S \]

---

**E.g.**

- **PCA facts**
  - \( F_1 \) to \( F_K \)
- **PCA res**
  - \( U_1 \) to \( U_S \)
- **stocks log.ret**
  - \( X_1 \) to \( X_S \)
- **stocks lin. rets**
  - \( R_1 \) to \( R_N \)
- **port ret**
  - \( R_w \)
- **attribute**
  - \( \sum_k d_k Z_k \)
- **GICS sectors**
  - \( Z_1 \) to \( Z_K \)

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1 joint scenario

**conditional scenarios given** \( X \)
Risk Estimation

1. Risk drivers estimation

\[ X_s = \sum_k b_{s,k} F_k + U_s \]

- \( X_s \) = risk driver
- \( b_{n,k} \) = loading
- \( F_k \) = dominant factor
- \( U_n \) = residual

2. Pricing

\[ R_n = g_n (X_1, \ldots, X_S) \]

3. Aggregation

\[ R_w = \sum_n w_n R_n \]

4. Portfolio risk estimation

\[ S\text{dev} \{ R_w \} = \text{exact} \]
\[ V\alpha R \{ R_w \} = \text{exact} \]

Risk Attribution

5. Attribution factors

\[ Z_k | X_1, \ldots, X_S \]

6. Portfolio risk attribution: top down

\[ R_w = \sum_k d_{w,k} Z_k + \eta_w \]

- \( R_w \) = portfolio return
- \( d_{w,k} \) = attribution loading
- \( Z_k \) = attribution factor
- \( \eta_w \) = residual

---

E.g.

- PCA facts
- PCA res
- stocks log.rets
- stocks lin. rets
- port ret
- attribution
- GICS sectors

1 joint scenario

X

conditional scenarios given \( X \)
### Risk Estimation

1. Risk drivers estimation

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- \( X_s \): risk driver
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2. Pricing

\[ R_n = g_n (X_1, \ldots, X_S) \]

3. Aggregation

\[ R_w = \sum_n w_n R_n \]

4. Portfolio risk estimation

- Standard deviation \( \text{std} \{ R_w \} \) = exact
- VaR \( \{ R_w \} \) = exact

### Risk Attribution

5. Attribution factors

\[ Z_k | X_1, \ldots, X_S \]

6. Portfolio risk attribution: top down

\[ R_w = \sum_k d_{w,k} Z_k + \eta_w \]

- \( R_w \): portfolio return
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- \( Z_k \): attribution factor
- \( \eta_w \): residual

---

### Diagram

- **E.g.** PCA facts, PCA res, stocks log.rets, stocks lin. rets, port ret, attribution, GICS sectors
- **1 joint scenario**
- **Conditional scenarios given** \( X \)
1. Risk drivers estimation
\[ X_s = \sum_k b_{s,k} F_k + U_s \]
- \( X_s \) = risk driver
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- \( F_k \) = dominant factor
- \( U_n \) = residual

2. Pricing
\[ R_n = g_n (X_1, \ldots, X_S) \]

3. Aggregation
\[ R_w = \sum_n w_n R_n \]

4. Portfolio risk estimation
- Variance \( \text{Var} \{ R_w \} \) = exact
- Standard deviation \( \text{StdDev} \{ R_w \} \) = exact

5. Attribution factors
\[ Z_k | X_1, \ldots, X_S \]

6. Portfolio risk attribution: top down
\[ R_w = \sum_k d_{w,k} Z_k + \eta_w \]
- \( R_w \) = portfolio return
- \( d_{w,k} \) = attribution loading
- \( Z_k \) = attribution factor
- \( \eta_w \) = residual

E.g. PCA facts | PCA res | stocks log.rets | stocks lin. rets | port ret | attribution | GICS sectors
\[ F_1 \quad F_K \quad U_1 \quad U_S \quad X_1 \quad X_S \quad R_1 \quad R_N \quad R_w \quad \sum_k d_{w,k} Z_k \quad Z_1 \quad Z_K \]

1 joint scenario

Conditional scenarios given \( X \)
**Risk Estimation**

1. Risk drivers estimation

\[ X_s = \sum_k b_{s,k} F_k + U_s \]
\[ X_s = \text{risk driver} \]
\[ b_{n,k} = \text{loading} \]
\[ F_k = \text{dominant factor} \]
\[ U_n = \text{residual} \]

2. Pricing

\[ R_n = g_n (X_1, \ldots, X_S) \]

3. Aggregation

\[ R_w = \sum_n w_n R_n \]

4. Portfolio risk estimation

- Sdev\{R_w\} = exact
- VaR\{R_w\} = exact

**Risk Attribution**

5. Attribution factors

\[ Z_k \mid X_1, \ldots, X_S \]

6. Portfolio risk attribution: top down

\[ R_w = \sum_k d_{w,k} Z_k + \eta_w \]
\[ d_{w} = \arg\max_{d \in \mathcal{C}} T (R_w, \sum_k d_k Z_k) \]
\[ R_w = \text{portfolio return} \]
\[ d_{w,k} = \text{attribution loading} \]
\[ Z_k = \text{attribution factor} \]
\[ \eta_w = \text{residual} \]
### Risk Estimation

1. **Risk drivers estimation**
   \[ X_s = \sum_k b_{s,k} F_k + U_s \]
   - \( X_s \) = risk driver
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   - \( F_k \) = dominant factor
   - \( U_n \) = residual

2. **Pricing**
   \[ R_n = g_n (X_1, \ldots, X_S) \]

3. **Aggregation**
   \[ R_w = \sum_n w_n R_n \]

4. **Portfolio risk estimation**
   \[ Sdev \{ R_w \} = \text{exact} \]
   \[ V a R \{ R_w \} = \text{exact} \]

### Risk Attribution

5. **Attribution factors**
   \[ Z_k | X_1, \ldots, X_S \]

6. **Portfolio risk attribution: top down**
   \[ R_w = \sum_k d_{w,k} Z_k + \eta_w \]
   - \( R_w \) = portfolio return
   - \( d_{w,k} \) = attribution loading
   - \( Z_k \) = attribution factor
   - \( \eta_w \) = residual

### Diagram

- PCA facts
- PCA res
- stocks log.rets
- stocks lin. rets
- port ret
- attribution
- GICS sectors

1 joint scenario

Conditional scenarios given \( X \)
**Risk Estimation**

1. Risk drivers estimation
   \[ X_s = \sum_k b_{s,k} F_k + U_s \]
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   - \( F_k \): dominant factor
   - \( U_n \): residual

2. Pricing
   \[ R_n = g_n (X_1, \ldots, X_S) \]

3. Aggregation
   \[ R_w = \sum_n w_n R_n \]

4. Portfolio risk estimation
   \[ \text{Var} \{ R_w \} = \text{exact} \]
   \[ \text{E} \{ R_w \} = \text{exact} \]

**Risk Attribution**

5. Attribution factors
   \[ \tilde{Z}_k | X_1, \ldots, X_S \]

6. Portfolio risk attribution: top down
   \[ R_w = \sum_k \tilde{d}_{w,k} \tilde{Z}_k + \eta_w \]
   - \( R_w \): portfolio return
   - \( \tilde{d}_{w,k} \): attribution loading
   - \( \tilde{Z}_k \): attribution factor
   - \( \eta_w \): residual

**FACTORS ON DEMAND**

- **Attilio Meucci**

**FOD – Theory**

- **Risk Estimation**
- **Risk Attribution**

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**Diagram**

- PCA facts
- PCA res
- stocks log.rets
- stocks lin. rets
- port ret
- attribution
- hedges

1 joint scenario

conditional scenarios given \( X \)
Risk Estimation

1. Risk drivers estimation
   \[ X_s = \sum_k b_{n,k} F_k + U_s \]
   - \( X_s \) = risk driver
   - \( b_{n,k} \) = loading
   - \( F_k \) = dominant factor
   - \( U_s \) = residual

2. Pricing
   \[ R_n = g_n (X_1, \ldots, X_S) \]

3. Aggregation
   \[ R_w = \sum_n w_n R_n \]

4. Portfolio risk estimation
   \[ \text{Sdev} \{ R_w \} = \text{exact} \]
   \[ \text{Var} \{ R_w \} = \text{exact} \]

Risk Attribution

5. Attribution factors
   \[ Z_k | X_1, \ldots, X_S \]

6. Portfolio risk attribution: top down
   \[ R_w = \sum_k d_{w,k} Z_k + \eta_w \]
   - \( R_w \) = portfolio return
   - \( d_{w,k} \) = attribution loading
   - \( Z_k \) = attribution factor
   - \( \eta_w \) = residual

Factors on Demand - Features
1. Estimation factors \( F \) and loadings \( b \) are chosen to optimize the explanation power
Risk Estimation
1. Risk drivers estimation
\[ X_s = \sum_k b_{s,k} F_k + U_s \]
2. Pricing
\[ R_n = g_n(X_1, \ldots, X_S) \neq \sum_s \delta_{n,s} X_s \]
3. Aggregation
\[ R_w = \sum_n w_n R_n \]
4. Portfolio risk estimation
\[
\begin{align*}
\text{Sdev \{R_w\}} & = \text{exact} \\
\text{Var \{R_w\}} & = \text{exact}
\end{align*}
\]

Risk Attribution
5. Attribution factors
\[ Z_k \mid X_1, \ldots, X_S \]
\[ R_n = \sum_k d_{n,k} Z_k + \eta_n \]
6. Portfolio risk attribution: top down
\[ R_w = \sum_k d_{w,k} Z_k + \eta_w \]
\[ d_{w} = \arg \max_{d \in \mathcal{C}} T(R_w, \sum_k d_k Z_k) \]
\[ R_w = \text{portfolio return} \]
\[ d_{w,k} = \text{attrition loading} \]
\[ Z_k = \text{attrition factor} \]
\[ \eta_w = \text{residual} \]

Factors on Demand - Features
1. Estimation factors \( F \) and loadings \( b \) are chosen to optimize the explanation power
2. Exact risk numbers through exact pricing
Risk Estimation

1. Risk drivers estimation
\[ X_s = \sum_k b_{n,k} F_k + U_s \]
   \( X_s \) = risk driver
   \( b_{n,k} \) = loading
   \( F_k \) = dominant factor
   \( U_s \) = residual

2. Pricing
\[ R_n = g_n (X_1, \ldots, X_S) \]

3. Aggregation
\[ R_w = \sum_n w_n R_n \]

4. Portfolio risk estimation
\[ \text{Sdev} \{ R_w \} = \text{exact} \]
\[ \text{Var} \{ R_w \} = \text{exact} \]

Risk Attribution

5. Attribution factors
\[ Z_k | X_1, \ldots, X_S \]

6. Portfolio risk attribution: top down
\[ R_w = \sum_k d_{w,k} Z_k + \eta_w \]
   \( R_w \) = portfolio return
   \( d_{w,k} \) = attribution loading
   \( Z_k \) = attribution factor
   \( \eta_w \) = residual

Factors on Demand - Features

1. Estimation factors \( F \) and loadings \( b \) are chosen to optimize the explanation power
2. Exact risk numbers through exact pricing
3. Attribution factors \( Z \) are chosen to be interpretable and practical for hedging
Risk Estimation
1. Risk drivers estimation
\[ X_s = \sum_k b_{n,k} F_k + U_s \]
\[ X_s = \text{risk driver} \]
\[ b_{n,k} = \text{loading} \]
\[ F_k = \text{dominant factor} \]
\[ U_s = \text{residual} \]

2. Pricing
\[ R_n = g_n (X_1, \ldots, X_S) \]

3. Aggregation
\[ R_w = \sum_n w_n R_n \]

4. Portfolio risk estimation
\[ \text{Std} \{ R_w \} = \text{exact} \]
\[ \text{Var} \{ R_w \} = \text{exact} \]

Risk Attribution
5. Attribution factors
\[ Z_k | X_1, \ldots, X_S \]

6. Portfolio risk attribution: top down
\[ R_w = \sum_k d_{w,k} Z_k + \eta_w \]
\[ R_w = \text{portfolio return} \]
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\[ Z_k = \text{attribution factor} \]
\[ \eta_w = \text{residual} \]

Factors on Demand - Features
1. Estimation factors \( F \) and loadings \( b \) are chosen to optimize the explanation power
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3. Attribution factors \( Z \) are chosen to be interpretable and practical for hedging
4. Attribution loadings \( d \) are chosen to optimize r-square, CVaR, downside risk, etc
### Risk Estimation

1. **Risk drivers estimation**
   \[
   X_s = \sum_k b_{s,k} F_k + U_s
   \]
   - \(X_s\) = risk driver
   - \(b_{n,k}\) = loading
   - \(F_k\) = dominant factor
   - \(U_s\) = residual

2. **Pricing**
   \[
   R_n = g_n (X_1, \ldots, X_S)
   \]

3. **Aggregation**
   \[
   R_w = \sum_n w_n R_n
   \]

4. **Portfolio risk estimation**
   \[
   \text{Sdev} \{R_w\} = \text{exact}
   \]
   \[
   \text{Var} \{R_w\} = \text{exact}
   \]

### Risk Attribution

5. **Attribution factors**
   \[
   Z_k | X_1, \ldots, X_S
   \]

6. **Portfolio risk attribution: top down**
   \[
   R_w = \sum_k d_{w,k} Z_k + \eta_w
   \]
   - \(R_w\) = portfolio return
   - \(d_{w,k}\) = attribution loading
   - \(Z_k\) = attribution factor
   - \(\eta_w\) = residual

### Factors on Demand - Features

1. Estimation factors \(F\) and loadings \(b\) are chosen to optimize the explanation power
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5. Constraints allow for long-only, best-few-out-of-many, etc
Risk Estimation

1. Risk drivers estimation
\[ X_s = \sum_k b_{n,k} F_k + U_s \]
   \( X_s \): risk driver
   \( b_{n,k} \): loading
   \( F_k \): dominant factor
   \( U_s \): residual

2. Pricing
\[ R_n = g_n (X_1, \ldots, X_S) \]

3. Aggregation
\[ R_w = \sum_n w_n R_n \]

4. Portfolio risk estimation
\[
\begin{align*}
\text{Sdev} \{ R_w \} &= \text{exact} \\
\text{Var} \{ R_w \} &= \text{exact}
\end{align*}
\]

Risk Attribution

5. Attribution factors
\[ Z_k | X_1, \ldots, X_S \]

6. Portfolio risk attribution: top down
\[
\begin{align*}
\overline{R_w} &= \sum_k d_{w,k} Z_k + \eta_w \\
\eta_w &= \text{residual}
\end{align*}
\]

7. Security-level attribution
\[ R_n = \sum_k d_{n,k} Z_k + \eta_n \]

Factors on Demand - Features

1. Estimation factors \( F \) and loadings \( b \) are chosen to optimize the explanation power
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5. Constraints allow for long-only, best-few-out-of-many, etc
6. Exact Linear interpretation/hedge of non-linear securities
Risk Estimation

1. Risk drivers estimation
\[ X_s = \sum_k b_{s,k} F_k + U_s \]

\( X_s \) = risk driver
\( b_{n,k} \) = loading
\( F_k \) = dominant factor
\( U_n \) = residual

2. Pricing
\[ R_n = g_n (X_1, \ldots, X_S) \]

3. Aggregation
\[ R_w = \sum_n w_n R_n \]

4. Portfolio risk estimation
\[ S\text{dev} \{R_w\} = \text{exact} \]
\[ V\text{aR} \{R_w\} = \text{exact} \]

Risk Attribution

5. Attribution factors
\[ Z_k | X_1, \ldots, X_S \]

6. Portfolio risk attribution: top down
\[ R_w = \sum_k d_{w,k} Z_k + \eta_w \]

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\( Z_k \) = attribution factor
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Factors on Demand - Features

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6. No linear relationship between \( Z \) and \( F \): connection created by conditional distribution
Risk Estimation

1. Risk drivers estimation
   \[ X_s = \sum_k b_{s,k} F_k + U_s \]
   \[ X_s = \text{risk driver} \]
   \[ b_{n,k} = \text{loading} \]
   \[ F_k = \text{dominant factor} \]
   \[ U_n = \text{residual} \]

2. Pricing
   \[ R_n = g_n (X_1, \ldots, X_S) \]

3. Aggregation
   \[ R_w = \sum_n w_n R_n \]

4. Portfolio risk estimation
   \[ \text{Sdev} \{ R_w \} = \text{exact} \]
   \[ \text{Var} \{ R_w \} = \text{exact} \]

Risk Attribution

5. Attribution factors
   \[ \tilde{Z}_k|X_1, \ldots, X_S \]

6. Portfolio risk attribution: top down
   \[ R_w = \sum_k \tilde{d}_{w,k} \tilde{Z}_k + \tilde{\eta}_w \]
   \[ R_w = \text{portfolio return} \]
   \[ d_{w,k} = \text{attribution loading} \]
   \[ Z_k = \text{attribution factor} \]
   \[ \eta_w = \text{residual} \]

Factors on Demand - Features

1. Estimation factors \( F \) and loadings \( b \) are chosen to optimize the explanation power
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4. Attribution loadings \( d \) are chosen to optimize r-square, CVaR, downside risk, etc
5. Constraints allow for long-only, best-few-out-of-many, etc
6. Exact Linear interpretation/hedge of non-linear securities
7. No linear relationship between \( Z \) and \( F \): connection created by conditional distribution
8. Conditional distribution -> one estimation method, several possible interpretations/hedges
### Risk Estimation
1. Risk drivers estimation
   \[ X_s = \sum_k b_{s,k} F_k + U_s \]
   - \( X_s \): risk driver
   - \( b_{n,k} \): loading
   - \( F_k \): dominant factor
   - \( U_s \): residual

2. Pricing
   \[ R_n = g_n (X_1, \ldots, X_S) \]

3. Aggregation
   \[ R_w = \sum_n w_n R_n \]

4. Portfolio risk estimation
   \[ Sdev \{ R_w \} = \text{exact} \]
   \[ VaR \{ R_w \} = \text{exact} \]

### Risk Attribution
5. Attribution factors
   \[ Z_k | X_1, \ldots, X_S \]

7. Security-level attribution
   \[ R_n = \sum_k d_{n,k} Z_k + \eta_n \]
   - \( R_w \): portfolio return
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   - \( \eta_w \): residual

### Factors on Demand - Features
1. Estimation factors \( F \) and loadings \( b \) are chosen to optimize the explanation power
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**Risk Estimation**

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   \[ X_s = \sum_k b_{n,k} F_k + U_s \]
   - \( X_s \) = risk driver
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   - \( U_n \) = residual

2. Pricing
   \[ R_n = g_n (X_1, \ldots, X_S) \]

3. Aggregation
   \[ R_w = \sum_n w_n R_n \]

4. Portfolio risk estimation
   \[ Sdev \{ R_w \} = \text{exact} \]
   \[ VAR \{ R_w \} = \text{exact} \]

**Risk Attribution**

5. Attribution factors
   \[ Z_k | X_1, \ldots, X_S \]
   \[ R_n = \sum_k d_{n,k} Z_k + \eta_n \]

6. Portfolio risk attribution: top down
   \[ R_w = \sum_k d_{w,k} Z_k + \eta_w \]
   - \( R_w \) = portfolio return
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   - \( Z_k \) = attribution factor
   - \( \eta_w \) = residual

**Factors on Demand - Features**

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8. Conditional distribution -> one estimation method, several possible interpretations/hedges
9. Systematic + idiosyncratic -> dominant + residual
10. Top-down attribution provides portfolio-specific best model
## Factors on Demand - Features

1. Estimation factors $F$ and loadings $b$ are chosen to optimize the explanation power.
2. Exact risk numbers through exact pricing.
3. Attribution factors $Z$ are chosen to be interpretable and practical for hedging.
4. Attribution loadings $d$ are chosen to optimize $r$-square, CVaR, downside risk, etc.
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6. Exact Linear interpretation/hedge of non-linear securities.
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### Risk Estimation

1. Risk drivers estimation
   \[ X_s = \sum_k b_{s,k} F_k + U_s \]
   - $X_s$ = risk driver
   - $b_{n,k}$ = loading
   - $F_k$ = dominant factor
   - $U_n$ = residual

2. Pricing
   \[ R_n = g_n (X_1, \ldots, X_S) \]

3. Aggregation
   \[ R_w = \sum_n w_n R_n \]

4. Portfolio risk estimation
   - $\text{Sdev} \{ R_w \}$ = exact
   - $\text{Var} \{ R_w \}$ = exact

### Risk Attribution

5. Attribution factors
   \[ Z_k | X_1, \ldots, X_S \]

6. Portfolio risk attribution: top down
   \[ R_w = \sum_k d_{w,k} Z_k + \eta_w \]
   - $R_w$ = portfolio return
   - $d_{w,k}$ = attribution loading
   - $Z_k$ = attribution factor
   - $\eta_w$ = residual

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Attilio Meucci

**FACTORS ON DEMAND**

**Risk Estimation**

1. Risk drivers estimation
   \[ X_s = \sum_k b_{s,k} F_k + U_s \]
   - $X_s$ = risk driver
   - $b_{n,k}$ = loading
   - $F_k$ = dominant factor
   - $U_n$ = residual

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   \[ R_n = g_n (X_1, \ldots, X_S) \]

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   \[ R_w = \sum_n w_n R_n \]

4. Portfolio risk estimation
   - $\text{Sdev} \{ R_w \}$ = exact
   - $\text{Var} \{ R_w \}$ = exact

**Risk Attribution**

5. Attribution factors
   \[ Z_k | X_1, \ldots, X_S \]

6. Portfolio risk attribution: top down
   \[ R_w = \sum_k d_{w,k} Z_k + \eta_w \]
   - $R_w$ = portfolio return
   - $d_{w,k}$ = attribution loading
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**Factors on Demand - Features**

1. Estimation factors $F$ and loadings $b$ are chosen to optimize the explanation power.
2. Exact risk numbers through exact pricing.
3. Attribution factors $Z$ are chosen to be interpretable and practical for hedging.
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5. Constraints allow for long-only, best-few-out-of-many, etc.
6. Exact Linear interpretation/hedge of non-linear securities.
7. No linear relationship between $Z$ and $F$: connection created by conditional distribution.
8. Conditional distribution -> one estimation method, several possible interpretations/hedges.
**Factors on Demand – Frequently Asked Questions**

Q: Why not run a regression of portfolio returns $R$ vs. attribution factors $Z$?
   A: $R$ and $Z$ are not necessarily “invariants”

Q: Why abandon “systematic + idiosyncratic” model?
   A: $U$ is where managers look for “alpha” factors -> $\Sigma X \neq b\Sigma Fb' + diag (\sigma_U^2)$
   A: otherwise we cannot merge irrelevant “systematic” factors with “idiosyncratic” residual to obtain more efficient attribution/hedging
   A: in powerful estimation approaches (PCA,RMT) residual $U$ is never idiosyncratic
   A: that model is not a consequence of APT/CAPM
## Risk Estimation

1. Risk drivers estimation
   \[ X_s = \sum_k b_{s,k} F_k + U_s \]
   - \( X_s \): risk driver
   - \( b_{s,k} \): loading
   - \( F_k \): dominant factor
   - \( U_s \): residual

2. Pricing
   \[ R_n = g_n (X_1, \ldots, X_S) \]

3. Aggregation
   \[ R_w = \sum_n w_n R_n \]

4. Portfolio risk estimation
   \[ \text{Sdev}\left\{ R_w \right\} = \text{exact} \]
   \[ \text{Var}\left\{ R_w \right\} = \text{exact} \]

## Risk Attribution

5. Attribution factors
   \[ Z_k | X_1, \ldots, X_S \]

6. Portfolio risk attribution: top down
   \[ R_w = \sum_k d_{w,k} Z_k + \eta_w \]
   - \( R_w \): portfolio return
   - \( d_{w,k} \): attribution loading
   - \( Z_k \): attribution factor
   - \( \eta_w \): residual

### Factors on Demand – Frequently Asked Questions

**Q:** Why should we not use delta approximation?

**A:** Risk of derivatives or non linear instruments at multi-day horizon is distorted
Risk Estimation
1. Risk drivers estimation
   \[ X_s = \sum_k b_{s,k} F_k + U_s \]
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2. Pricing
   \[ R_n = g_n (X_1, \ldots, X_S) \]

3. Aggregation
   \[ R_w = \sum_n w_n R_n \]

4. Portfolio risk estimation
   \[ \text{Sdev} \{ R_w \} = \text{exact} \]
   \[ \text{VaR} \{ R_w \} = \text{exact} \]

Risk Attribution
5. Attribution factors
   \[ Z_k | X_1, \ldots, X_S \]

6. Portfolio risk attribution: top down
   \[ R_w = \sum_k d_{w,k} Z_k + \eta_w \]
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   - \( \eta_w \): residual

Factors on Demand – Frequently Asked Questions
Q: Do we have to generate conditional scenarios for \( Z \)?
   A: Not always: if using historical scenarios, use historical (drivers for) \( Z \)

Q: Does FOD recommend specific estimation/attribution factors/techniques?
   A: No, FOD proposes a flexible, modular methodology that hosts all techniques

Q: Does FOD dismiss traditional multi-purpose factor models
   A: No, all traditional model are special cases of FOD
Risk Estimation

1. Risk drivers estimation
   \[ X_s = \sum_k b_{s,k} F_k + U_s \]
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2. Pricing
   \[ R_n = g_n (X_1, \ldots, X_S) \]

3. Aggregation
   \[ R_w = \sum_n w_n R_n \]

4. Portfolio risk estimation
   \[
   \begin{align*}
   SD\{R_w\} &= \text{exact} \\
   VaR\{R_w\} &= \text{exact}
   \end{align*}
   \]

Risk Attribution

5. Attribution factors
   \[ Z_k X_1, \ldots, X_S \]

6. Portfolio risk attribution: top down
   \[
   \begin{align*}
   R_w &= \sum_k d_{w,k} Z_k + \eta_w \\
   d_{w,k} &= \arg\max_{d\in C} T (R_w, \sum_k d_k Z_k) \\
   R_w &= \text{portfolio return} \\
   d_{w,k} &= \text{attrtribution loading} \\
   Z_k &= \text{attrtribution factor} \\
   \eta_w &= \text{residual}
   \end{align*}
   \]

b, F: high statistical power
- Principal Component Analysis and Random Matrix Theory can be applied
- Factors and loadings are determined to minimize estimation error although they might be difficult to interpret.

Z: high interpretability/tradability
- Attribution factors examples
  - GICS Sectors: Material, Technology, Financials
  - Macro: S&P500, 10 year yield, Gold price, MSCI EM Index, Russell 2000

FOD Application #1: Optimize Factor Choice for Risk and Portfolio Mgmt.

FOD Application #1: Optimize Factor Choice for Risk and Portfolio Mgmt.
### Risk Estimation
1. Risk drivers estimation
   \[ X_s = \sum_k b_{s,k} F_k + U_s \]
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   \[ \text{Sdev} \{ R_w \} = \text{exact} \]
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### Risk Attribution
5. Attribution factors
   \[ (Z_k) X_1, \ldots, X_S \]

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   - \( Z_k \): attribution factor
   - \( \eta_w \): residual

FOD Application #1:
Optimize Factor Choice for Risk and Portfolio Mgmt.

- **b, F** : high statistical power
- **Z** : high interpretability/tradability

**volatility decomposition – per industry (RMT)**

**volatility decomposition – per factor (GICS)**
### Risk Estimation
1. Risk drivers estimation
\[
X_s = \sum_k b_{n,k} F_k + U_s
\]
- \(X_s\) = risk driver
- \(b_{n,k}\) = loading
- \(F_k\) = dominant factor
- \(U_s\) = residual

2. Pricing
\[
R_n = g_n (X_1, \ldots, X_S)
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4. Portfolio risk estimation
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\text{Sdev} \{R_w\} = \text{exact}
\]
\[
\text{Var} \{R_w\} = \text{exact}
\]

### Risk Attribution
5. Attribution factors
\[
Z_k X_1, \ldots, X_S
\]

6. Portfolio risk attribution: top down
\[
R_w = \sum_k d_{w,k} Z_k + \eta_w
\]
- \(R_w\) = portfolio return
- \(d_{w,k}\) = attribution loading
- \(Z_k\) = attribution factor
- \(\eta_w\) = residual

### FOD Application #2:
Custom attribution factors on the fly

- **X**: historical
  - No factor modes for \(X\), pure historical realization of risk drivers
  - \(R\) is not the time series of the returns
  - Explicitly no idiosyncratic term

- **Z**: \(g(X)\)
  - Attribution factors are deterministic functions of risk drivers
  - For instance, \(Z\) can be user-supplied definitions of value/momentum factors
  - FOD then allows to compare in real time the attribution to different, user-supplied factor models \(Z\) and \(\tilde{Z}\)
  - All models share the same risk statistics
Attilio Meucci

**Risk Estimation**

1. Risk drivers estimation
   \[ X_s = \sum_k b_{s,k} F_k + U_s \]
   - \( X_s \) = risk driver
   - \( b_{n,k} \) = loading
   - \( F_k \) = dominant factor
   - \( U_n \) = residual

2. Pricing
   \[ R_n = g_n (X_1, \ldots, X_S) \]

3. Aggregation
   \[ R_w = \sum_n w_n R_n \]

4. Portfolio risk estimation
   \[ \text{Sdev} \left\{ R_w \right\} = \text{exact} \]
   \[ \text{Var} \left\{ R_w \right\} = \text{exact} \]

**Risk Attribution**

5. Attribution factors
   \[ [Z_k] X_1, \ldots, X_S \]

6. Portfolio risk attribution: top down
   \[ R_w = \sum_k d_{w,k} Z_k + \eta_w \]
   - \( R_w \) = portfolio return
   - \( d_{w,k} \) = attribution loading
   - \( Z_k \) = attribution factor
   - \( \eta_w \) = residual

**FOD Application #3: Integrated Global and Regional Risk Models**

- Global factors \( Z \) are deterministic, linear functions (aggregations) of the regional factors
  \[ Z \equiv A \begin{pmatrix} F^{(\alpha)} \\ \vdots \\ F^{(\omega)} \end{pmatrix} \]

- Regional factors \( F \) constructed by cross-sectional regression on given loadings \( b \)
  - e.g. US Model: US sector factors
    \[ R^{(\alpha)} = B^{(\alpha)} F^{(\alpha)} + U^{(\alpha)} \]
  - e.g. UK Model: UK financial, UK utilities,...
    \[ R^{(\omega)} = B^{(\omega)} F^{(\omega)} + U^{(\omega)} \]

- e.g. global financial, global utilities,...
**FACTORS ON DEMAND**
Attilio Meucci

### Risk Estimation

1. Risk drivers estimation
   \[ X_s = \sum_k b_{s,k} F_k + U_s \]
   \( X_s \) = risk driver
   \( b_{s,k} \) = loading
   \( F_k \) = dominant factor
   \( U_s \) = residual

2. Pricing
   \[ R_n = g_n (X_1, \ldots, X_S) \]

3. Aggregation
   \[ R_w = \sum_n w_n R_n \]

4. Portfolio risk estimation
   \[
   \begin{align*}
   Sdev \{ R_w \} &= \text{exact} \\
   VaR \{ R_w \} &= \text{exact}
   \end{align*}
   \]

### Risk Attribution

5. Attribution factors
   \[ Z_k X_1, \ldots, X_S \]

6. Portfolio risk attribution: top down
   \[ R_w = \sum_k d_{w,k} Z_k + \eta_w \]
   \( R_w \) = portfolio return
   \( d_{w,k} \) = attribution loading
   \( Z_k \) = attribution factor
   \( \eta_w \) = residual

7. Security-level attribution
   \[ R_n = \sum_k d_{n,k} Z_k + \eta_n \]

---

**Z**: returns of hedging instruments; **\( d \)**: attribution target as CVaR

- For hedging, the attribution factors must be the linear returns \( Z = P(t+1)/P(t)-1 \) of tradables
- Linear attribution (6) is important for hedging: only portfolios, i.e. linear combinations, are traded
- Profits and losses of hedged p&l \( \eta \) play a non-symmetrical role: non-linear pricing (2) properly induces asymmetries on \( R \); downside target CVaR in (6) accounts for asymmetries in \( \eta \)
- Thus FOD hedging (full-pricing/CVaR) and Black-Scholes hedging (delta/r-square) are different

---

**Example:** units of underlying to hedge call options

<table>
<thead>
<tr>
<th></th>
<th>100 days</th>
<th>150 days</th>
<th>200 days</th>
<th>250 days</th>
<th>300 days</th>
</tr>
</thead>
<tbody>
<tr>
<td>FOD</td>
<td>5.8</td>
<td>5.3</td>
<td>5.0</td>
<td>4.9</td>
<td>4.8</td>
</tr>
<tr>
<td>BS</td>
<td>5.7</td>
<td>5.4</td>
<td>5.2</td>
<td>5.1</td>
<td>5.0</td>
</tr>
</tbody>
</table>
Risk Estimation

1. Risk drivers estimation
   \[ X_s = \sum_k b_{s,k} F_k + U_s \]
   - \( X_s \): risk driver
   - \( b_{s,k} \): loading
   - \( F_k \): dominant factor
   - \( U_s \): residual

2. Pricing
   \[ R_n = g_n(X_1, \ldots, X_S) \]

3. Aggregation
   \[ R_w = \sum_n w_n R_n \]

4. Portfolio risk estimation
   \[ \text{SDev}\{R_w\} = \text{exact} \]
   \[ \text{V}aR\{R_w\} = \text{exact} \]

Risk Attribution

5. Attribution factors
   \[ Z_k | X_1, \ldots, X_S \]

6. Portfolio risk attribution: top down
   \[ R_w = \sum_k d_{w,k} Z_k + \eta_w \]
   - \( R_w \): portfolio return
   - \( d_{w,k} \): attribution loading
   - \( Z_k \): attribution factor
   - \( \eta_w \): residual

7. Security-level attribution
   \[ R_n = \sum_k d_{n,k} Z_k + \eta_n \]

**FOD Application #5: Best Pool on Demand / flexible constraints**

**d: constraint “few relevant out of many” in top-down attribution**

- For hedging, traders prefer to put on fewer hedges. Therefore the selection of the best few trades should be optimized.
- For factor modeling, it does not make sense to include minimally represented factors in analysis. Better to add them to residual.
- Other constraints can be added (e.g. long only, sum-to-one, etc.)
### Risk Estimation

1. Risk drivers estimation
\[ X_s = \sum_k b_{s,k} F_k + U_s \]
   - \( X_s \): risk driver
   - \( b_{n,k} \): loading
   - \( F_k \): dominant factor
   - \( U_n \): residual

2. Pricing
\[ R_n = g_n (X_1, \ldots, X_s) \]

3. Aggregation
\[ R_w = \sum_n w_n R_n \]

4. Portfolio risk estimation
\[
\begin{align*}
\text{Sdev} \{ R_w \} &= \text{exact} \\
\text{Var} \{ R_w \} &= \text{exact}
\end{align*}
\]

### Risk Attribution

5. Attribution factors
\[ (Z_k) X_1, \ldots, X_S \]

6. Portfolio risk attribution: top down
\[ R_w = \sum_k d_{w,k} Z_k + \eta_w \]
   - \( R_w \): portfolio return
   - \( d_{w,k} \): attribution loading
   - \( Z_k \): attribution factor
   - \( \eta_w \): residual

7. Security-level attribution
\[ R_n = \sum_k d_{n,k} Z_k + \eta_n \]

---

**Z**: returns of sub-portfolios; portfolios: past holdings

- The attribution of the current holdings to the past holdings allows the portfolio manager to evaluate the turnover (half-life) of their positions

\[
\begin{align*}
Z_1 &= w_{t-1} R \\
\vdots \\
Z_K &= w_{t-K} R
\end{align*}
\]
### Risk Estimation

1. Risk drivers estimation

\[ X_s = \sum_k b_{s,k} F_k + U_s \]

- \( X_s \): risk driver
- \( b_{s,k} \): loading
- \( F_k \): dominant factor
- \( U_s \): residual

2. Pricing

\[ R_n = g_n (X_1, \ldots, X_S) \]

3. Aggregation

\[ R_w = \sum_n w_n R_n \]

4. Portfolio risk estimation

\[ \begin{align*}
\text{Sdev} \{R_w\} &= \text{exact} \\
\text{Var} \{R_w\} &= \text{exact}
\end{align*} \]

### Risk Attribution

5. Attribution factors

\[ (Z_k) X_1, \ldots, X_S \]

6. Portfolio risk attribution: top down

\[ R_w = \sum_k d_{w,k} Z_k + \eta_w \]

- \( R_w \): portfolio return
- \( d_{w,k} \): attribution loading
- \( Z_k \): attribution factor
- \( \eta_w \): residual

7. Security-level attribution

\[ R_n = \sum_k d_{n,k} Z_k + \eta_n \]

### FOD Application #6: Turnover-trading persistence

- The attribution of the current holdings to the past holdings allows portfolio managers to evaluate the turnover (half-life) of their positions.

- If the attribution target in (6) is set as the r-square and the attribution optimization is unconstrained we obtain the analytical solution in Grinold (2006):

\[ d_w = (W' \Sigma_R W)^{-1} W' \Sigma_R w_t \]

- FOD allows portfolio managers to customize their analysis, with arbitrary targets and constraints:

\[ Z = \begin{bmatrix} w'_{t-1} R \\ \vdots \\ w'_{t-K} R \end{bmatrix} \]
**Risk Estimation**

1. Risk drivers estimation

\[ X_s = \sum_k b_{s,k} F_k + U_s \]

- \( X_s \): risk driver
- \( b_{n,k} \): loading
- \( F_k \): dominant factor
- \( U_n \): residual

2. Pricing

\[ R_n = g_n (X_1, \ldots, X_S) \]

3. Aggregation

\[ R_w = \sum_n w_n R_n \]

4. Portfolio risk estimation

- \( \text{Sdev} \{ R_w \} = \text{exact} \)
- \( \text{Var} \{ R_w \} = \text{exact} \)

**Risk Attribution**

5. Attribution factors

\[ Z_k X_1, \ldots, X_S \]

6. Portfolio risk attribution: top down

\[ R_w = \sum_k d_{w,k} Z_k + \eta_w \]

- \( R_w \): portfolio return
- \( d_{w,k} \): attribution loading
- \( Z_k \): attribution factor
- \( \eta_w \): residual

\[ d' = 1, d \geq 0 \]

**FOD Applications #7: Point in Time Style Analysis**

- **Z**: style factors; constraints: long-only, sum-to-one

- Traditional style analysis a-la-Sharpe runs a constrained regression of portfolio returns \( R_p(t) \) on style factors \( Z(t) \)
- In traditional style analysis the past returns are affected by the past allocation decisions \( R_p(t-k)=w(t-k) \times R(t-k) \) includes a component due to rebalancing \( w(t-k) \)
- FOD allows to perform point-in-time style analysis based only the current exposures \( w(t) \)
Attilio Meucci - “Factors on Demand”
*Risk*, July 2010, p 84-89

MATLAB Central Files Exchange (see above article)

www.symmys.com > Teaching > Talks