Attilio Meucci

Managing Diversification

COMMON MEASURES OF DIVERSIFICATION

DIVERSIFICATION DISTRIBUTION

MEAN-DIVERSIFICATION FRONTIER

CONDITIONAL ANALYSIS

REFERENCES

Managing diversification: common measures of diversification

$$R_{\mathbf{w}} \equiv \mathbf{w}'\mathbf{R}$$

portfolio return

returns of securities (stocks, bonds, options, structured products, …)

portfolio weights

Managing diversification: common measures of diversification

$$R_{\mathbf{w}} \equiv \mathbf{w}'\mathbf{R}.$$

weight-based definitions

$$\mathcal{D}_{Her} \equiv 1 - \mathbf{w}'\mathbf{w}.$$

portfolio weights

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distribution

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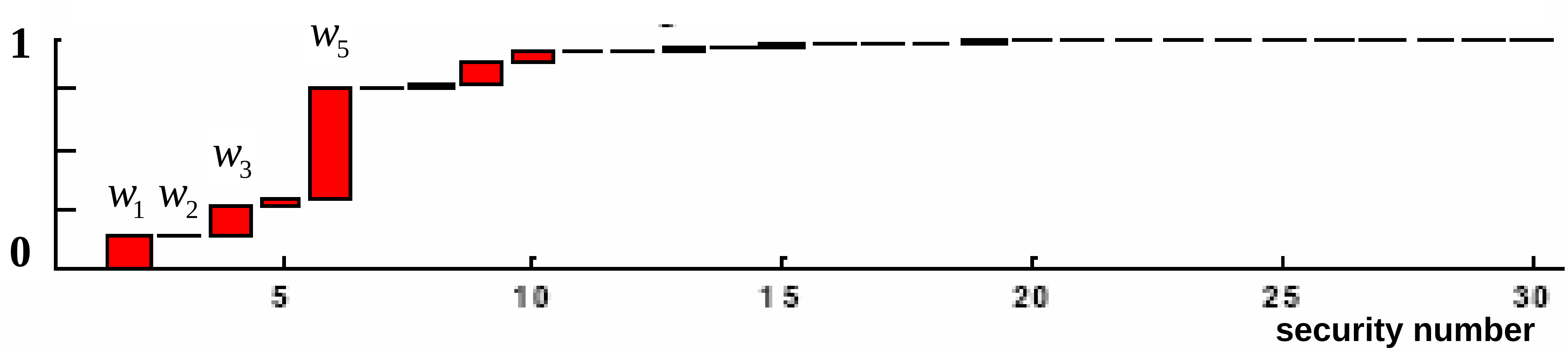
portfolio weights

- positive
- sum to one



Managing diversification: common measures of diversification

$$R_{\mathbf{w}} \equiv \mathbf{w}'\mathbf{R}$$

weight-based definitions

$$\mathcal{D}_{Her} \equiv 1 - \mathbf{w}'\mathbf{w}$$

$$\mathcal{D}_{BP} \equiv -\sum_{n=1}^{N} w_n \ln(w_n)$$ **entropy**

$\Leftarrow$ **distribution**

$\Updownarrow$

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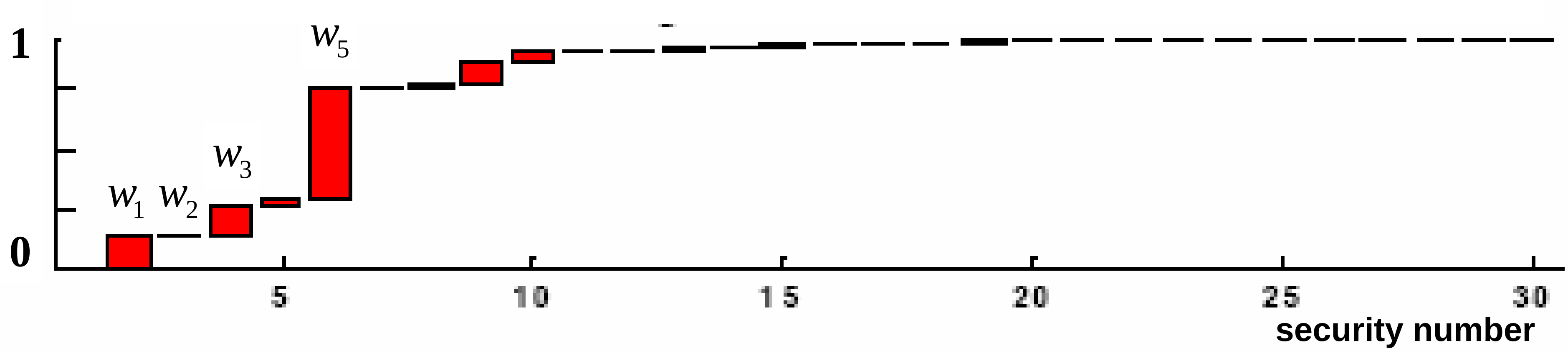

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$$\mathcal{D}_{HK}^{(\gamma)} \equiv -\left(\sum_{n=1}^{N} w_n^{\gamma}\right)^{\frac{1}{\gamma-1}}$$

$\Leftarrow$ **distribution**

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- positive

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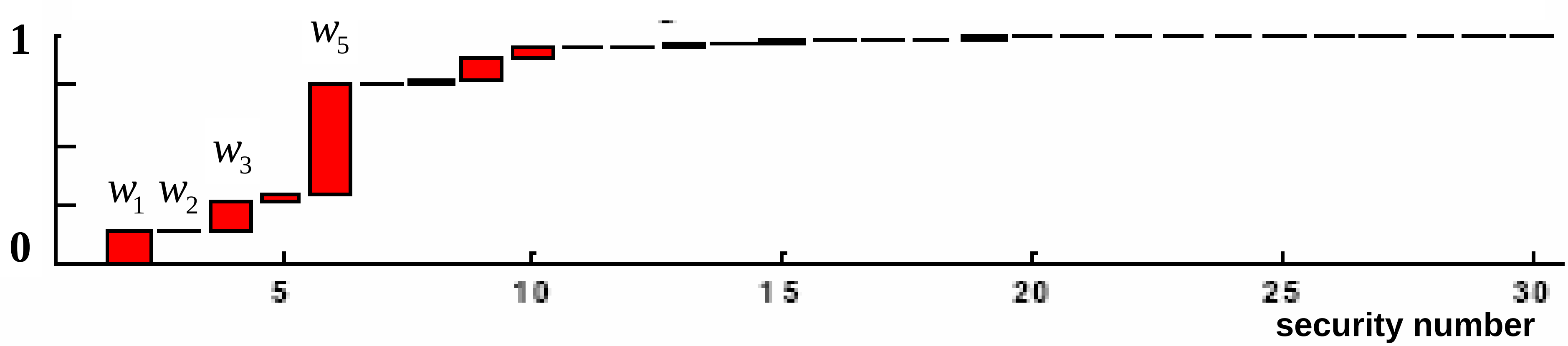

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risk-based definitions

$$\mathcal{D}_{IP} \equiv 1 - \mathbf{w}'\mathbf{C}\mathbf{w},$$

returns correlation matrix

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risk-based definitions

$$\mathcal{D}_{IP} \equiv 1 - \mathbf{w}'\mathbf{C}\mathbf{w},$$

$$\mathcal{D}_{Dif} \equiv \boldsymbol{\sigma}'\mathbf{w} - \sqrt{\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}}.$$

returns standard deviations ($\boldsymbol{\sigma}$)

returns covariance matrix ($\boldsymbol{\Sigma}$)

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factor-based definition

$$R_n \equiv \sum_{k=1}^{K} \beta_{n,k} F_k + \epsilon_n$$

$$\mathcal{D}_{IS} \equiv 1 - \frac{\text{Var}\{R_\epsilon\}}{\text{Var}\{R_{\mathbf{w}}\}}$$

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These definitions apply in specific circumstances and or under restrictive hypotheses

Managing diversification – diversification distribution

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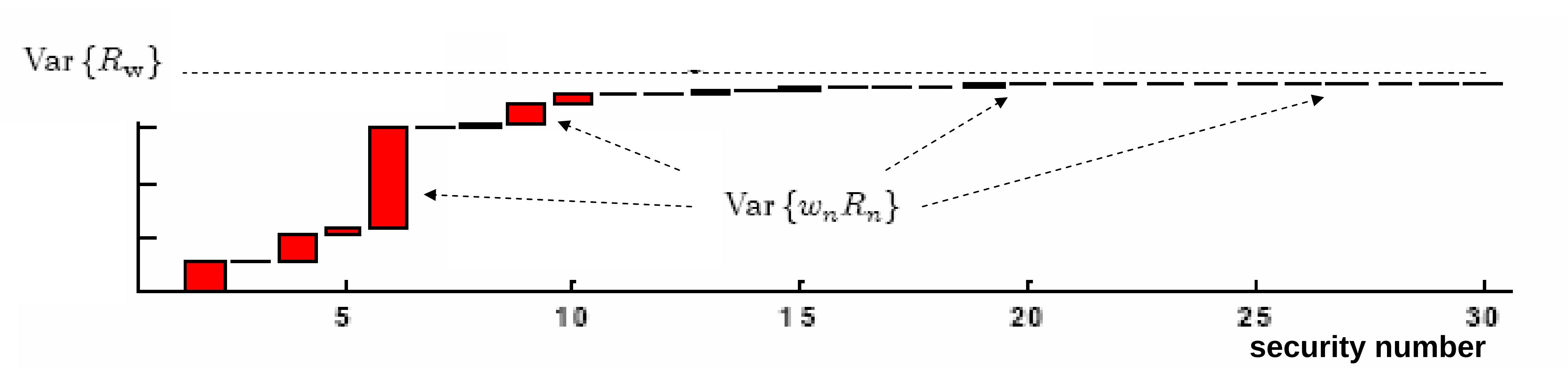
$$\mathrm{Var}\{R_{\mathbf{w}}\} \equiv \sum_{n=1}^{N} \mathrm{Var}\{w_n R_n\}$$

if correlations = 0



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if correlations = 0

Example: portfolio of two securities

- one bond $w_1 = 50\%$ $\mathrm{Var}\{R_1\} = (1\%)^2$

- one stock $w_2 = 50\%$ $\mathrm{Var}\{R_2\} = (30\%)^2$

weighs highly diversified

risk highly concentrated

Managing diversification – diversification distribution

$$R_{\mathbf{w}} \equiv \mathbf{w}'\mathbf{R}$$

$$\mathrm{Var}\{R_{\mathbf{w}}\} \not\equiv \sum_{n=1}^{N} \mathrm{Var}\{w_n R_n\}$$

if correlations ≠ 0

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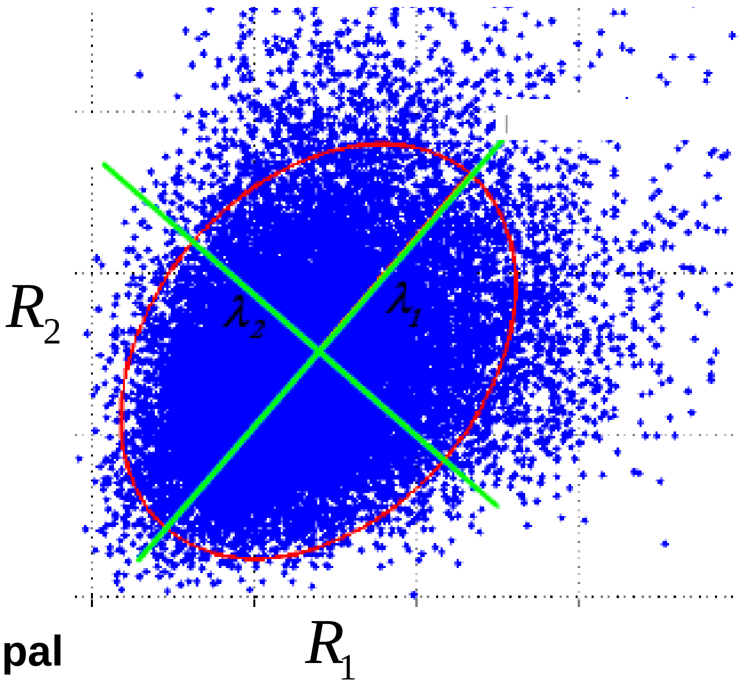

$$\Sigma \equiv \mathrm{Cov}\{\mathbf{R}\}$$

$\downarrow$

$$\mathbf{\Sigma} \equiv \mathbf{E}\,\mathbf{\Lambda}\,\mathbf{E}'$$

PCA

$$\mathbf{E} \equiv (\mathbf{e}_1, \ldots, \mathbf{e}_N)$$

eigenvectors

$\Updownarrow$

principal portfolios

principal portfolio 2

principal portfolio 1

$$\mathbf{\Lambda} \equiv \mathrm{diag}(\lambda_1^2, \ldots, \lambda_N^2)$$

eigenvalues

$\Updownarrow$

principal variances

$$\lambda_n^2 \equiv \mathrm{Var}\{\mathbf{e}_n'\mathbf{R}\}$$

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$\widetilde{\mathbf{R}} \equiv \mathbf{E}^{-1}\mathbf{R}$ return of principal portfolios

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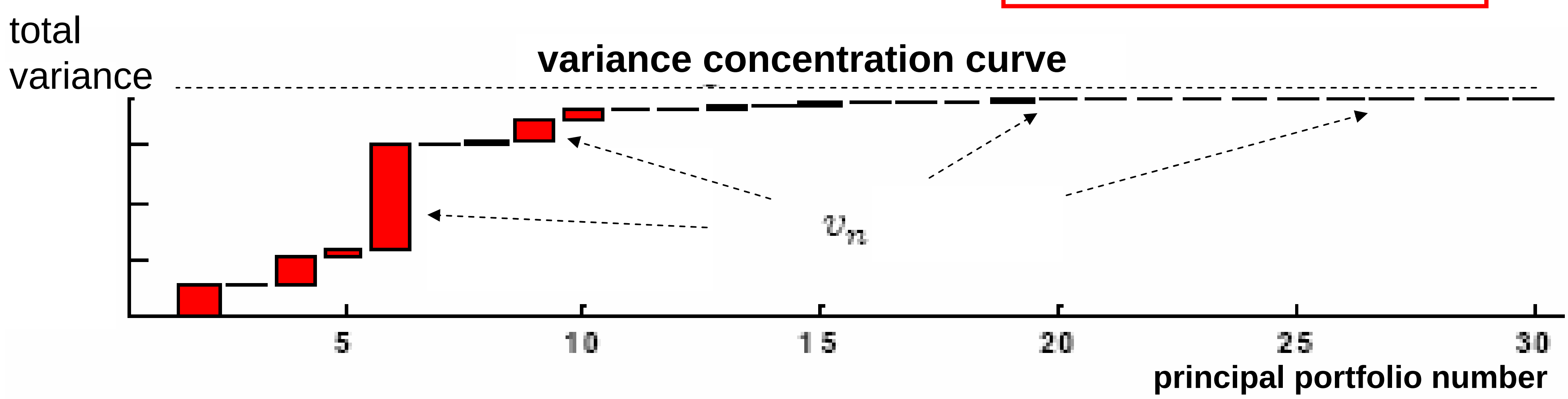

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$$R_{\mathbf{w}} \equiv \widetilde{\mathbf{w}}'\widetilde{\mathbf{R}}$$

$v_n \equiv \widetilde{w}_n^2 \lambda_n^2$ variance concentration curve

contribution to original portfolio variance from n-th principal portfolio:

$$\mathrm{Var}\{R_{\mathbf{w}}\} \equiv \sum_{n=1}^{N} v_n$$

Managing diversification – diversification distribution

$$R_{\mathbf{w}} \equiv \mathbf{w}'\mathbf{R}.$$

$$\text{Var}\{R_{\mathbf{w}}\} \not\equiv \sum_{n=1}^{N} \text{Var}\{w_n R_n\}$$

Example: portfolio of two government bonds in same duration bucket

Bond 1 $w_1 = 50\%$ $\text{Var}\{R_1\} = (1\%)^2$

Bond 2 $w_2 = 50\%$ $\text{Var}\{R_2\} = (1\%)^2$

weighs highly diversified

volatility homegeneous

$\tilde{\mathbf{R}} \equiv \mathbf{E}^{-1}\mathbf{R}$ return of principal portfolios

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volatility homegeneous

concentration curve concentrated on 1 principal portfolio

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contribution to original portfolio variance from n-th principal portfolio:

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Managing diversification – diversification distribution

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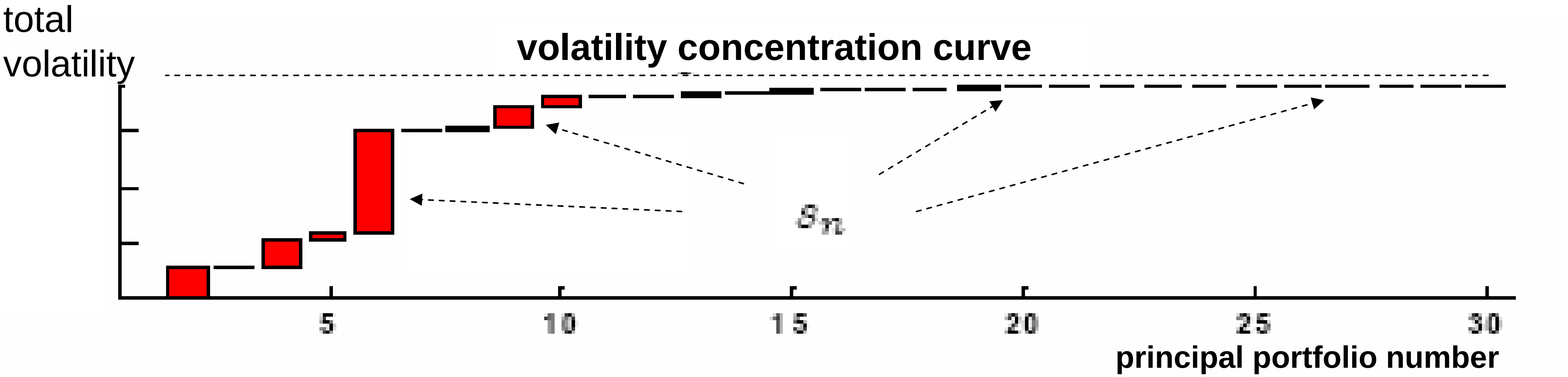

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$s_n \equiv \frac{\widetilde{w}_n^2 \lambda_n^2}{\mathrm{Sd}\{R_{\mathbf{w}}\}}$ volatility concentration curve

contribution to original portfolio volatility from n-th principal portfolio: "hot spots"

Managing diversification – diversification distribution

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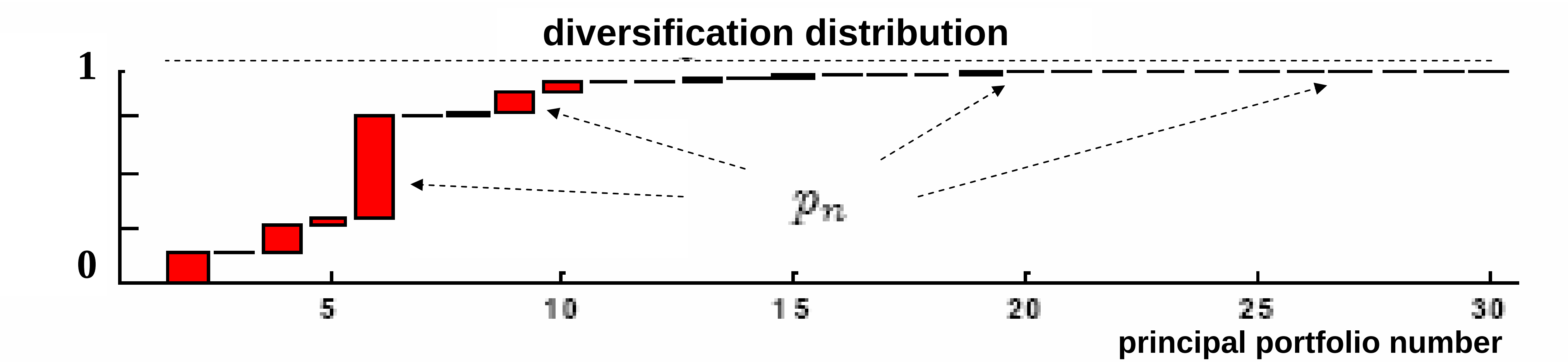

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$\Updownarrow$

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$\Updownarrow$

$p_n \equiv \dfrac{\widetilde{w}_n^2 \lambda_n^2}{\mathrm{Var}\{R_{\mathbf{w}}\}}$ diversification distribution

contribution to original portfolio r-square from n-th principal portfolio

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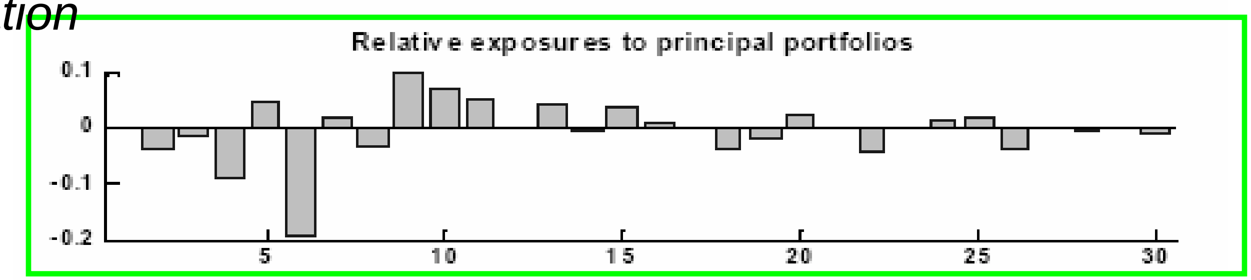
Managing diversification

$$\mathbf{w} \mapsto \mathbf{w} - \mathbf{b}$$



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$$\begin{cases} v_n \equiv \widetilde{w}_n^2 \lambda_n^2 & \text{variance concentration curve} \\ & \Updownarrow \\ s_n \equiv \dfrac{\widetilde{w}_n^2 \lambda_n^2}{\operatorname{Sd}\{R_{\mathbf{w}}\}} & \text{volatility concentration curve} \\ & \Updownarrow \\ p_n \equiv \dfrac{\widetilde{w}_n^2 \lambda_n^2}{\operatorname{Var}\{R_{\mathbf{w}}\}} & \text{diversification distribution} \end{cases}$$

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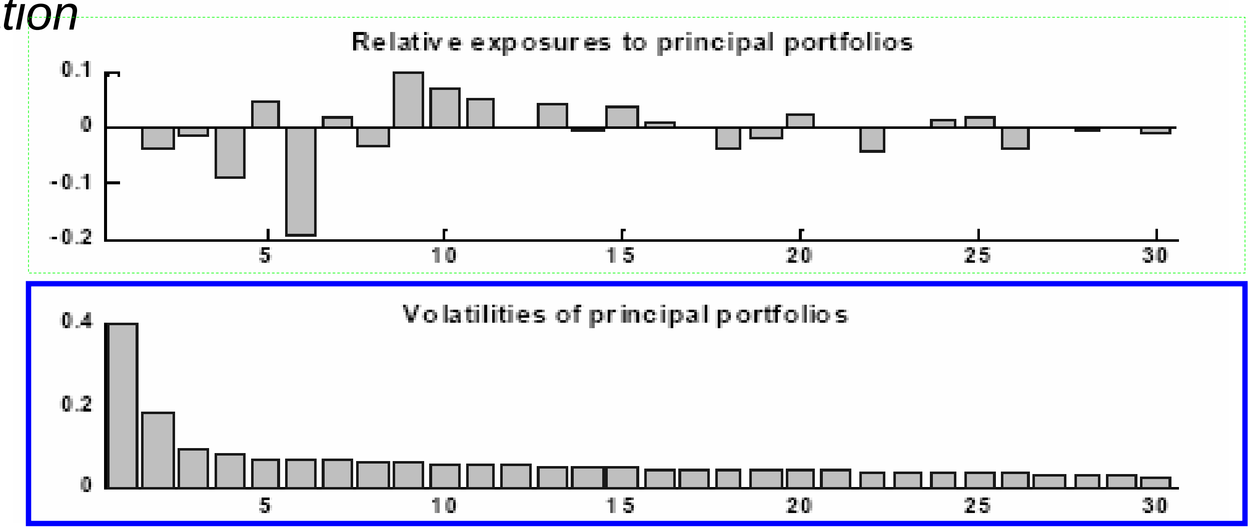

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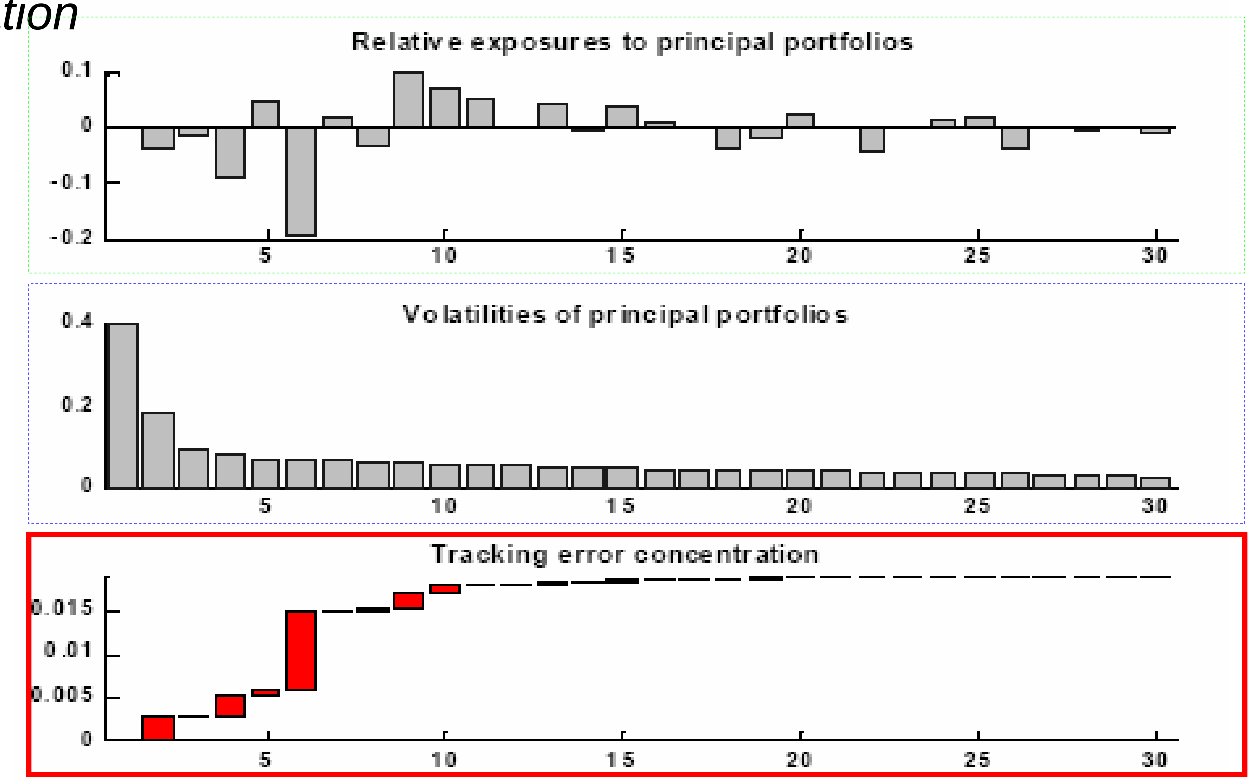

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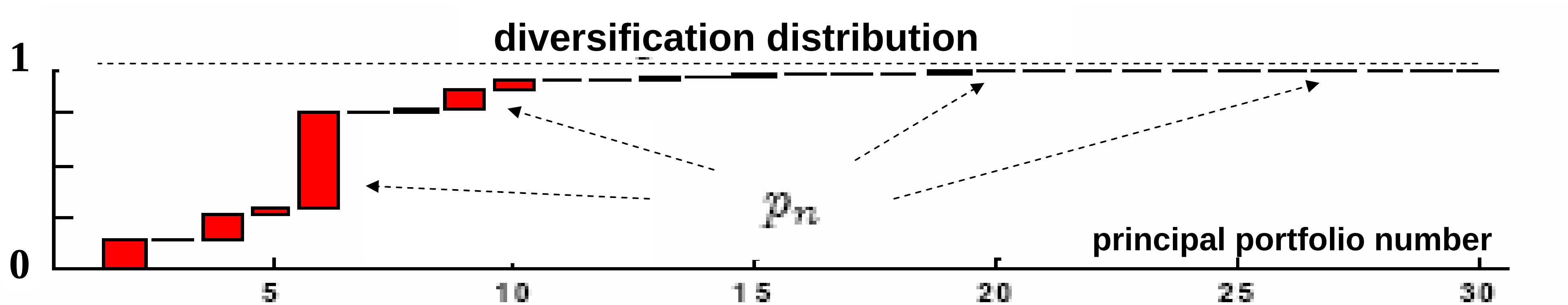
Managing diversification – mean-diversification frontier



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Managing diversification – mean-diversification frontier

entropy: $-\sum_{n=1}^{N} p_n \ln p_n$

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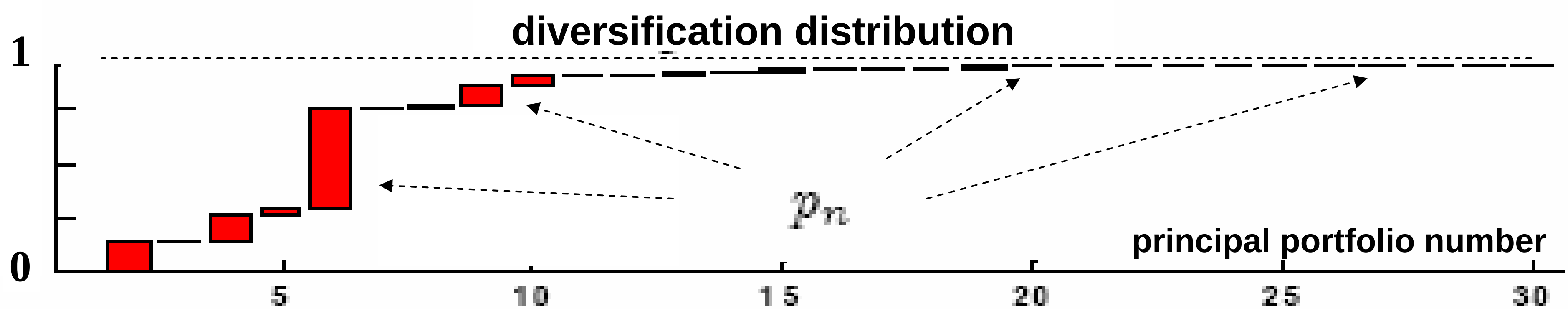
diversification



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Managing diversification – mean-diversification frontier

effective number of bets

$$\mathcal{N}_{Ent} \equiv \exp\left(-\sum_{n=1}^{N} p_n \ln p_n\right)$$

⇕

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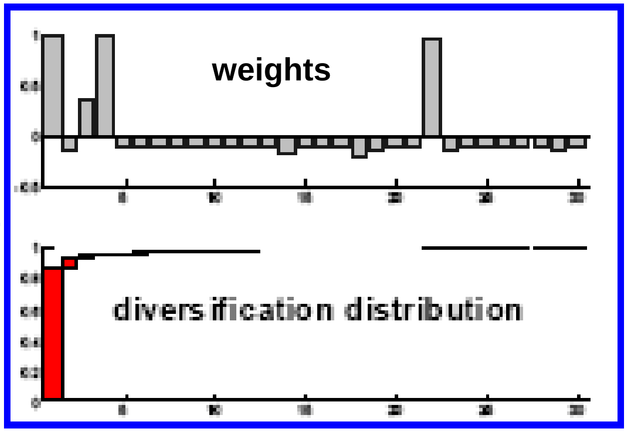
full concentration $\mathcal{N}_{Ent} \approx 1$



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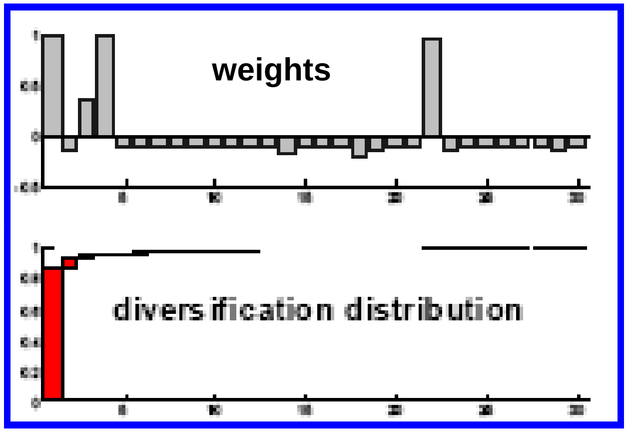

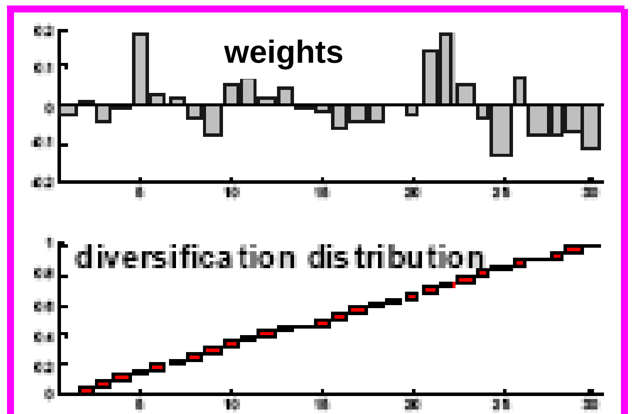

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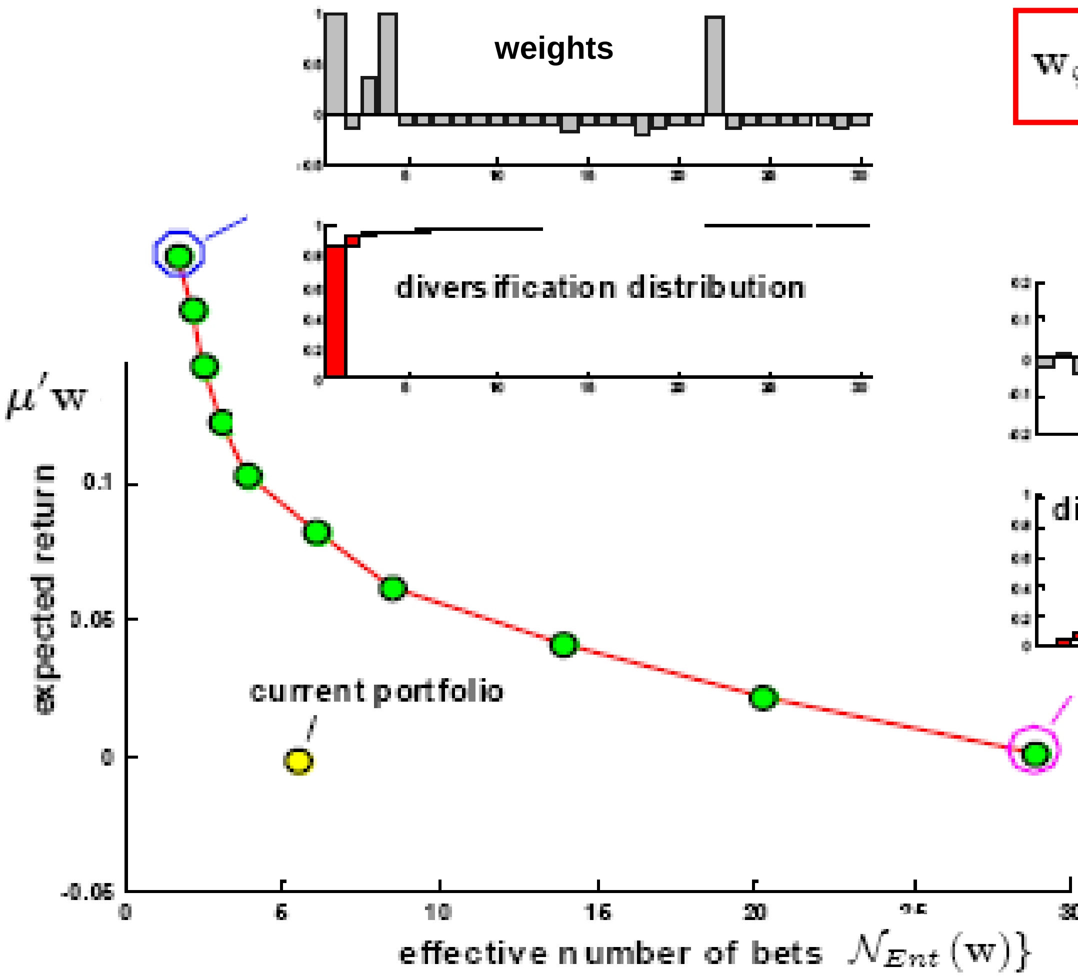
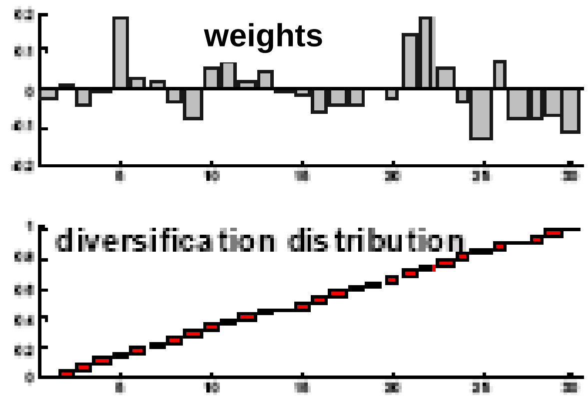
full diversification $\mathcal{N}_{Ent} \approx N$

mean-diversification frontier

$$\mathbf{w}_{\varphi} \equiv \underset{\mathbf{w} \in \mathcal{C}}{\operatorname{argmax}} \left\{ \varphi \mu' \mathbf{w} + (1-\varphi) \mathcal{N}_{Ent}(\mathbf{w}) \right\}$$





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full diversification $\mathcal{N}_{Ent} \approx N$.

mean-diversification frontier

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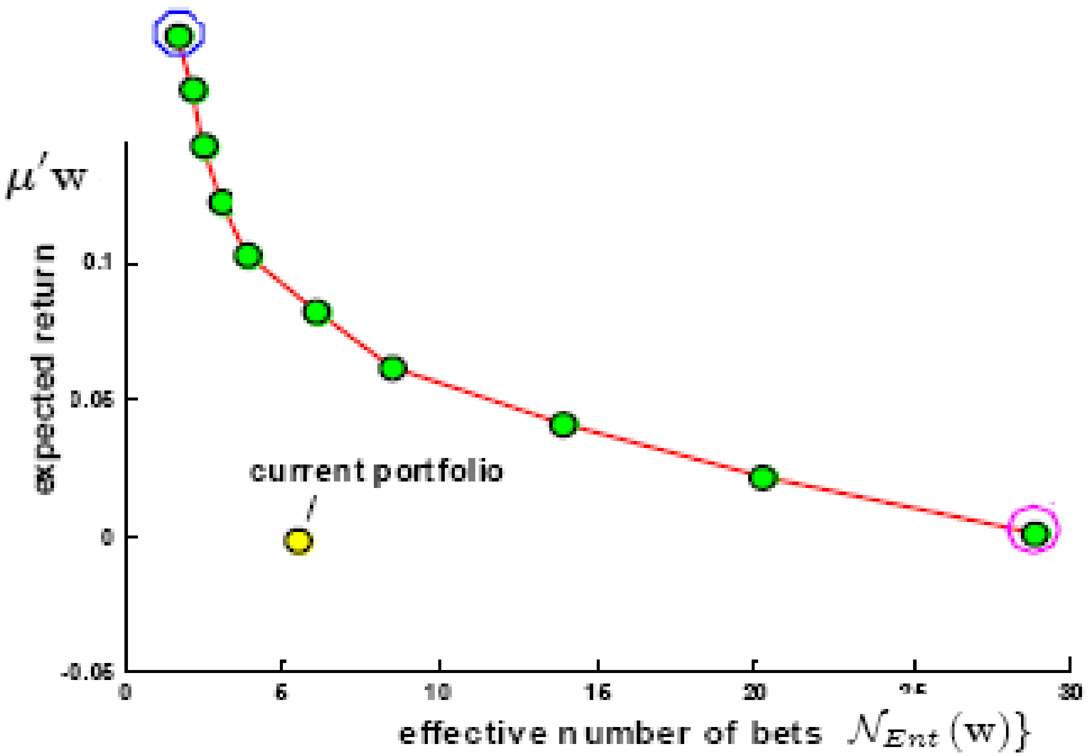

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transaction costs

$$\mu'\mathbf{w} \mapsto \mu'\mathbf{w} - \mathcal{T}(\mathbf{w}, \mathbf{w}_{cur})$$

$$\mathbf{w}_{\varphi} \equiv \underset{\mathbf{w}\in\mathcal{C}}{\operatorname{argmax}} \left\{\varphi\mu'\mathbf{w} + (1-\varphi)\mathcal{N}_{Ent}(\mathbf{w})\right\}$$

Managing diversification – mean-diversification frontier

effective number of bets

$$\mathcal{N}_{Ent} \equiv \exp\left(-\sum_{n=1}^{N} p_n \ln p_n\right)$$

full concentration $\mathcal{N}_{Ent} \approx 1$

full diversification $\mathcal{N}_{Ent} \approx N$.

transaction costs adjusted mean-diversification frontier

$$\mathbf{w}_{\varphi} \equiv \underset{\mathbf{w} \in \mathcal{C}}{\operatorname{argmax}} \{\varphi\left(\mu'\mathbf{w} - \mathcal{T}\left(\mathbf{w}, \mathbf{w}_{cur}\right)\right) + (1-\varphi)\,\mathcal{N}_{Ent}\left(\mathbf{w}\right)\}$$

Managing diversification – mean-diversification frontier

effective number of bets

$$\mathcal{N}_{Ent} \equiv \exp\left(-\sum_{n=1}^{N} p_n \ln p_n\right)$$

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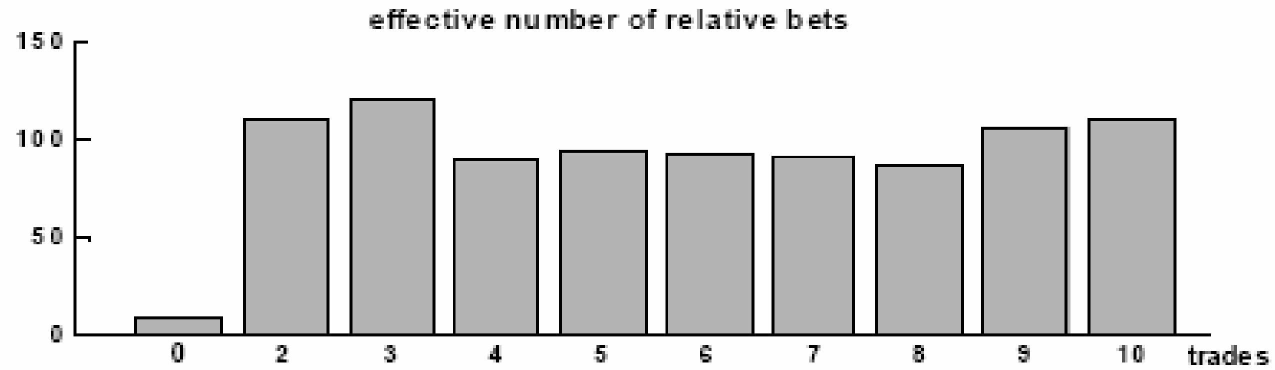
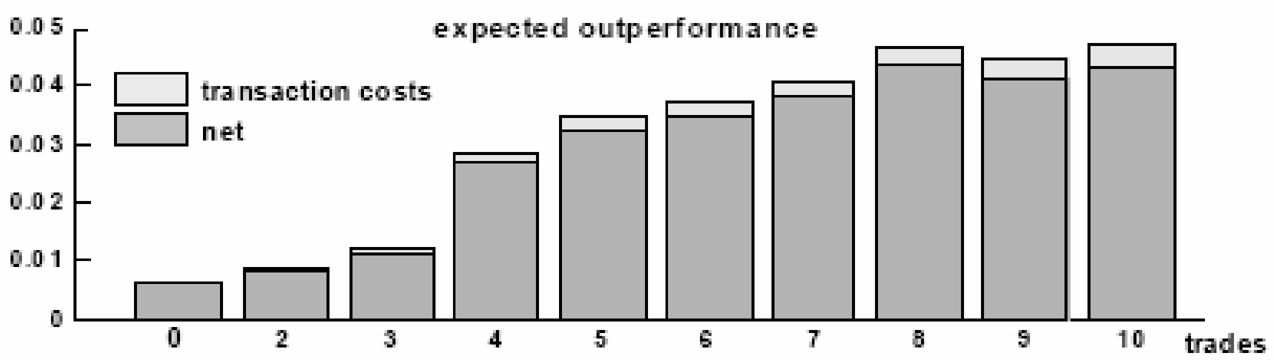
transaction costs adjusted mean-diversification frontier

$$\mathbf{w}_{\varphi} \equiv \underset{\mathbf{w} \in \mathcal{C}}{\operatorname{argmax}} \left\{ \varphi \left(\boldsymbol{\mu}' \mathbf{w} - \mathcal{T}(\mathbf{w}, \mathbf{w}_{cur}) \right) + (1 - \varphi)\, \mathcal{N}_{Ent}(\mathbf{w}) \right\}$$





COMMON MEASURES OF DIVERSIFICATION

DIVERSIFICATION DISTRIBUTION

MEAN-DIVERSIFICATION FRONTIER

CONDITIONAL ANALYSIS

REFERENCES

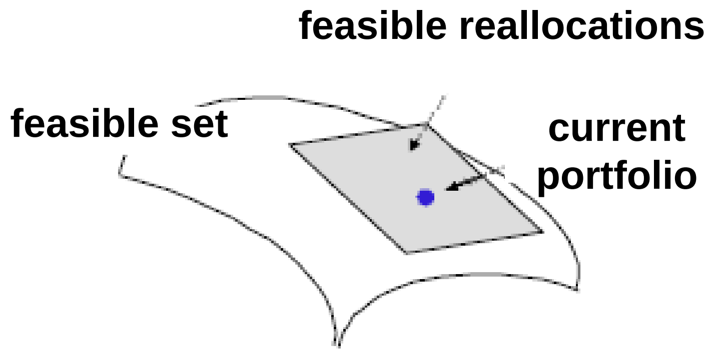
Managing diversification – conditional analysis

constraints

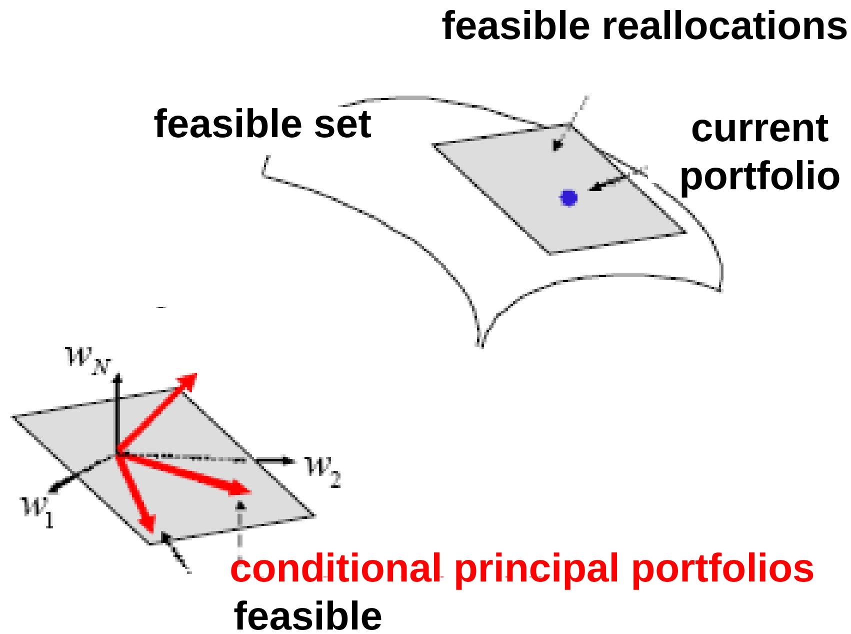
$$\mathbf{A}\Delta\mathbf{w} \equiv \mathbf{0}$$

$\mathbf{A}$: $K \times N$

$\Delta\mathbf{w}$: $N \times 1$



Managing diversification – conditional analysis



constraints

$$\boxed{\mathbf{A}\Delta\mathbf{w} \equiv \mathbf{0}}$$

conditional PCA

Feasible trades

$$n = K+1, \ldots, N$$

$$\mathbf{e}_n \equiv \underset{\mathbf{e}'\mathbf{e}\equiv 1}{\operatorname{argmax}} \{\mathbf{e}'\boldsymbol{\Sigma}\mathbf{e}\}$$

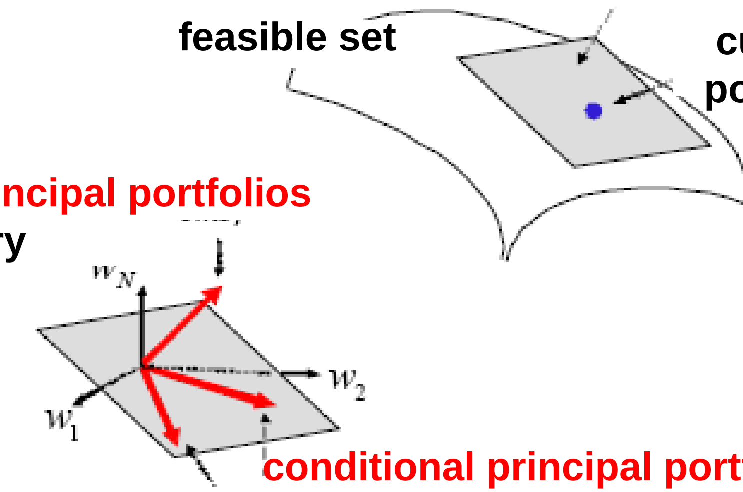
such that

$$\left\{ \begin{array}{l} \mathbf{e}'\boldsymbol{\Sigma}\mathbf{e}_j \equiv 0 \\ \text{for all existing } \mathbf{e}_j \\ \boxed{\mathbf{A}\mathbf{e} \equiv \mathbf{0}} \end{array} \right.$$

Managing diversification – conditional analysis

constraints

$$\mathbf{A}\Delta\mathbf{w} \equiv \mathbf{0}$$



conditional PCA

Feasible trades

$n = K+1, \ldots, N$

$$\mathbf{e}_n \equiv \underset{\mathbf{e}'\mathbf{e}\equiv 1}{\operatorname{argmax}} \{\mathbf{e}'\Sigma\mathbf{e}\}$$

such that $\begin{cases} \mathbf{e}'\Sigma\mathbf{e}_j \equiv 0 \\ \text{for all existing } \mathbf{e}_j \\ \mathbf{A}\mathbf{e} \equiv \mathbf{0} \end{cases}$

Complementary, unfeasible trades

$n = 1, \ldots, K$

$$\mathbf{e}_n \equiv \underset{\mathbf{e}'\mathbf{e}\equiv 1}{\operatorname{argmax}} \{\mathbf{e}'\Sigma\mathbf{e}\}$$

such that $\begin{cases} \mathbf{e}'\Sigma\mathbf{e}_j \equiv 0 \\ \text{for all existing } \mathbf{e}_j \end{cases}$

- **Article:**
  Attilio Meucci, "Managing Diversification"
  ***Risk* - May 2009**
  extended version available at http://ssrn.com/abstract=1358533

- **MATLAB examples:**
  MATLAB Central Files Exchange

- **This presentation:**
  www.symmys.com > Teaching > Talks