

# Capital Markets and Portfolio Theory

Master's in Mathematical Finance  
Courant Institute of Mathematical Sciences – New York University  
Spring semester 2006

## Instructor

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## Synopsis

The course covers quantitative portfolio management and risk management from the foundations to the most advanced developments.

Multivariate statistics and estimation methods are analyzed. These include non-parametric, maximum-likelihood under non-normal hypotheses, shrinkage, robust, and general Bayesian techniques. Implicit factor models, such as principal component analysis, and explicit factor models, such as the CAPM and the APT, are introduced. Evaluation methods such as stochastic dominance, expected utility, value at risk and coherent measures are discussed in a unified setting and applied in a variety of contexts, including prospect theory, total return and benchmark allocation. Classical portfolio optimization is introduced in a general setting and then analyzed in the sub-optimal two-step mean-variance framework, as well as in the mean-CVaR framework. Optimization under estimation risk is then discussed: the Black-Litterman approach, more general Bayesian approaches, the resampling procedure and robust optimization techniques.

The course consists of theory and applications. The theory follows closely the textbook *Risk and Asset Allocation* by A. Meucci. The applications are implemented in MATLAB<sup>®</sup>. They are displayed interactively during the course to support intuition and are further analyzed by the students in their homework. No previous knowledge of this software is assumed.

Prerequisites: probability, multivariate calculus and linear algebra

Suggested reading: *Risk and Asset Allocation* by A. Meucci – Springer (2005)

Course webpage: [symmys.com](http://symmys.com) > Teaching > Courses

## Lecture 1 – 01/18/06: univariate statistics

### Contents

- Course overview
- Introduction to MATLAB
  - Set path and work from command window
  - Generate scripts
  - Generate functions
  - Debug
  - Plot 2D (lines, scatter plots, histograms)
  - Plot 3D (surfaces, histograms)
- Representations of univariate distributions
  - Probability density function
  - Cumulative distribution function
  - Quantile
  - Characteristic function
- Distribution of transformations of random variables
  - Invertible transformations
  - Positive affine transformations
- Summary statistics:
  - Location
  - Scale
  - Higher moments
- Taxonomy of univariate distributions
  - Uniform distribution
  - Normal distribution
  - Cauchy distribution
  - Student t distribution
  - Gamma distribution
  - Lognormal distribution
  - Empirical distribution

### References: (!) = mandatory, (?) = optional

- A. Meucci, *Risk and Asset Allocation* – Springer:
  - (!): 1.1->1.3; (?): Preface
- A. Meucci, *Risk and Asset Allocation, Technical Appendices* – symmys.com:
  - (!): wherever cited above; (?): otherwise
- (!): Support material for Lecture 1 – symmys.com
- (?): A. Meucci, *Risk and Asset Allocation, Exercise Book* – symmys.com

## Lecture 2 – 01/25/06: multivariate statistics I

### Contents

- Representations of multivariate distributions
  - Probability density function
  - Cumulative distribution function
  - Characteristic function
  - Regularization
- Glivenko-Cantelli theorem and Monte Carlo simulations
  - Empirical cdf
  - Empirical quantile by interpolation
  - Histograms and pdf
- Copula-marginal factorization
  - Marginals as histograms
  - Grades
  - Copula representation via pdf and cdf
  - Co-monotonic random variables
  - Copula simulation
- Simulation of generic distributions via copula and quantile

### References: (!) = mandatory, (?) = optional

- A. Meucci, *Risk and Asset Allocation* – Springer:
  - (!): 2.1, 2.2, 4.2 only p.178-179, B.1->B.5
- A. Meucci, *Risk and Asset Allocation, Technical Appendices* – symmys.com:
  - (!): wherever cited above; (?): otherwise
- (!): Support material for Lecture 2 – symmys.com
- (?): A. Meucci, *Risk and Asset Allocation, Exercise Book* – symmys.com

## Lecture 3 – 02/01/06: multivariate statistics II

### Contents

- Conditional distribution and independence
- Taxonomy of multivariate distributions
  - Normal distribution
  - Cauchy distribution
  - Student t distribution
  - Log-distributions
- Dependence and concordance summary statistics
  - Special copulas
  - Schweizer-Wolff measure
  - Kendall tau
  - Spearman rho

### References: (!) = mandatory, (?) = optional

- A. Meucci, *Risk and Asset Allocation* – Springer:
  - (!): 2.3->2.5, 2.6.2->2.6.5, 2.7, A.5; (?): A.1->A.4
- A. Meucci, *Risk and Asset Allocation, Technical Appendices* – symmys.com:
  - (!): wherever cited above; (?): otherwise
- (!): Support material for Lecture 3 – symmys.com
- (?): A. Meucci, *Risk and Asset Allocation, Exercise Book* – symmys.com

## Lecture 4 – 02/08/06: multivariate statistics III

### Contents

- Shape summary statistics
  - Affine equivariance of shape statistics
  - Expected value – covariance
  - Mode – modal dispersion
- Location-dispersion ellipsoid
  - Spectral theorem
  - Statistical interpretation
- Pearson correlation
- Taxonomy of multivariate distributions
  - Uniform distribution
  - Wishart distribution
  - Empirical distribution
  - Order statistics
  - Elliptical distributions
  - Stable distributions
  - Infinitely divisible distributions

### References: (!) = mandatory, (?) = optional

- A. Meucci, *Risk and Asset Allocation* – Springer:
  - (!): 2.4->2.7
- A. Meucci, *Risk and Asset Allocation, Technical Appendices* – symmys.com:
  - (!): wherever cited above; (?): otherwise
- (!): Support material for Lecture 4 – symmys.com
- (?): A. Meucci, *Risk and Asset Allocation, Exercise Book* – symmys.com

## Lecture 5 – 02/15/06: market modeling I

### Contents

- Review of order statistics
- Review of location-dispersion ellipsoid
- The quest for invariance
  - Equities: log-returns
  - Fixed-income: changes in yield to maturity
  - Derivatives: changes in at-the-money implied volatility

References: (!) = mandatory, (?) = optional

- A. Meucci, *Risk and Asset Allocation* – Springer:
  - (!): Review + 3.1 (including derivatives)
- A. Meucci, *Risk and Asset Allocation, Technical Appendices* – symmys.com:
  - (!): wherever cited above; (?): otherwise
- (!): Support material for Lecture 5 – symmys.com
- (?): A. Meucci, *Risk and Asset Allocation, Exercise Book* – symmys.com

## Lecture 6 – 02/22/06: market modeling II

### Contents

- Review of functional analysis (convolution, Fourier transform, etc.)
- Projection of invariants to the investment horizon
  - Analytical: characteristic function
  - Numerical: pdf and FFT
- Pricing of invariants at the investment horizon
  - Analytical: log-distributions for raw securities
  - Numerical: Monte Carlo
  - Approximate: theta-delta/vega-gamma
  - Approximate: carry-duration-convexity

References: (!) = mandatory, (?) = optional

- A. Meucci, *Risk and Asset Allocation* – Springer:
  - (!): Review (A.1->A.5, B.1->B.4), 3.2, 3.3
- A. Meucci, *Risk and Asset Allocation, Technical Appendices* – symmys.com:
  - (!): wherever cited above; (?): otherwise
- (!): Support material for Lecture 6 – symmys.com
- (?): A. Meucci, *Risk and Asset Allocation, Exercise Book* – symmys.com

## Lecture 7 – 03/01/06: market modeling III

### Contents

- Dimension reduction, theory:
  - Linear regression
  - Principal component analysis
- Dimension reduction, notable examples
  - Capital Asset Pricing Model
  - Arbitrage Pricing Theory
  - Fama-French factors
- Principal component analysis of the swap market

References: (!) = mandatory, (?) = optional

- A. Meucci, *Risk and Asset Allocation* – Springer:
  - (!): 3.4, 3.5
- A. Meucci, *Risk and Asset Allocation, Technical Appendices* – symmys.com:
  - (!): wherever cited above; (?): otherwise
- (!): Support material for Lecture 7 – symmys.com
- (?): A. Meucci, *Risk and Asset Allocation, Exercise Book* – symmys.com

## Lecture 8 – 03/08/06: estimation I

### Contents

- Estimators
  - general definitions
  - evaluation: bias, inefficiency, error
- Non-parametric estimators
  - Sample quantile and order statistics.
  - Sample mean/covariance and best-fitting ellipsoid
  - Sample factor loadings and ordinary least squares
- Maximum-likelihood estimators
  - Normal hypothesis: sample estimators
  - Non-normal hypothesis: outlier rejection

References: (!) = mandatory, (?) = optional

- A. Meucci, *Risk and Asset Allocation* – Springer:
  - (!): 4.1, 4.2, 4.3
- A. Meucci, *Risk and Asset Allocation, Technical Appendices* – symmys.com:
  - (!): wherever cited above; (?): otherwise
- (!): Support material for Lecture 8 – symmys.com
- (?): A. Meucci, *Risk and Asset Allocation, Exercise Book* – symmys.com

## Lecture 9 – 03/22/06: estimation II

### Contents

- Shrinkage estimators
  - Stein mean
  - Ledoit-Wolf covariance
- Robust estimators
  - Assessing robustness: the influence function
  - Huber's "M" robust estimators
  - Outlier detection and high-breakdown estimators
  - Minimum-volume ellipsoid and minimum-covariance determinant

### References: (!) = mandatory, (?) = optional

- A. Meucci, *Risk and Asset Allocation* – Springer:
  - (!): 4.4, 4.5, 4.6
- A. Meucci, *Risk and Asset Allocation, Technical Appendices* – symmys.com:
  - (!): wherever cited above; (?): otherwise
- (!): Support material for Lecture 9 – symmys.com
- (?): A. Meucci, *Risk and Asset Allocation, Exercise Book* – symmys.com

## Lecture 10 – 03/29/06: portfolio evaluation I

### Contents

- Investor's objectives
  - Total return
  - Benchmark allocation
  - Net profits
- Global evaluation of a portfolio: stochastic dominance
- Summary evaluation of a portfolio: indices of satisfaction
  - Money-equivalence
  - Estimability
  - Sensibility
  - Consistence with stochastic dominance
  - Constancy
  - Positive homogeneity
  - Translation invariance
  - Sub- and super-additivity
  - Co-monotonic additivity
  - Concavity and convexity
- Risk aversion, risk propensity and risk neutrality
- Expected utility and certainty-equivalent

References: (!) = mandatory, (?) = optional

- A. Meucci, *Risk and Asset Allocation* – Springer:
  - (!): 5.1, 5.2, 5.3, 5.4
- A. Meucci, *Risk and Asset Allocation, Technical Appendices* – symmys.com:
  - (!): wherever cited above; (?): otherwise
- (!): Support material for Lecture 10 – symmys.com
- (?): A. Meucci, *Risk and Asset Allocation, Exercise Book* – symmys.com

## Lecture 11 – 04/05/06: portfolio evaluation II

### Contents

- Quantiles and value at risk (VaR)
  - Properties
  - Analytical solutions
  - Numerical approximations
- Coherent measures of performance
  - Expected shortfall (ES) and conditional value at risk (CVaR)
  - Spectral measures of performance

References: (!) = mandatory, (?) = optional

- A. Meucci, *Risk and Asset Allocation* – Springer:
  - (!): 5.5, 5.6
- A. Meucci, *Risk and Asset Allocation, Technical Appendices* – symmys.com:
  - (!): wherever cited above; (?): otherwise
- (!): Support material for Lecture 11 – symmys.com
- (?): A. Meucci, *Risk and Asset Allocation, Exercise Book* – symmys.com

## Lecture 12 – 04/12/06: portfolio optimization I

### Contents

- The general framework: a guided tour
- Constrained optimization: computationally tractable problems
  - Linear and quadratic programming
  - Second order and semi-definite cone programming
- Mean-variance optimization
  - Analytical solutions: total return vs. benchmark allocation
  - Numerical solutions: quadratic programming
  - Pitfalls of the mean-variance approach

References: (!) = mandatory, (?) = optional

- A. Meucci, *Risk and Asset Allocation* – Springer:
  - (!): 6.1 -> 6.5, 6.7
- A. Meucci, *Risk and Asset Allocation, Technical Appendices* – symmys.com:
  - (!): wherever cited above; (?): otherwise
- (!): Support material for Lecture 12 – symmys.com
- (?): A. Meucci, *Risk and Asset Allocation, Exercise Book* – symmys.com

## Lecture 13 – 04/19/06: portfolio optimization II - Bayesian statistics

### Contents

- Benchmark vs. total-return portfolio management
  - Mean-variance approximation
  - Analytical solutions in total-return coordinates
  - Analytical solutions in relative-return coordinates:  
expected outperformance, tracking error, information ratio
- Bayesian estimators
  - General theory
  - Analytically tractable examples

### References: (!) = mandatory, (?) = optional

- A. Meucci, *Risk and Asset Allocation* – Springer:  
(!): 6.6, 7.1, 7.2
- A. Meucci, *Risk and Asset Allocation, Technical Appendices* – symmys.com:  
(!): wherever cited above; (?): otherwise
- (!): Support material for Lecture 13 – symmys.com
- (?): A. Meucci, *Risk and Asset Allocation, Exercise Book* – symmys.com

## Lecture 14 – 04/26/06: portfolio optimization with estimation risk

### Contents

- Allocations as decisions
  - Opportunity cost
  - Allocation decisions evaluated as estimators
- Prior allocation
- Sample-based allocation
  - Error in satisfaction and constraint assessment
  - Leverage of estimation risk
- Bayesian allocation
  - Predictive return allocation
  - Classical-equivalent allocation
- Black-Litterman allocation
  - Views on market parameters
  - Views on the market realizations
- Copula-opinion pooling allocation
- Resampled allocation
- Robust allocation
  - Second-order cone programming problems
  - Semi-definite programming problems
- Robust Bayesian allocation

References: (!) = mandatory, (?) = optional

- A. Meucci, *Risk and Asset Allocation* – Springer:
  - (?): 8.1 -> 9.5
- A. Meucci, *Risk and Asset Allocation, Technical Appendices* – symmys.com:
  - (?): wherever cited above
- (?): Support material for Lecture 14 – symmys.com
- (?): A. Meucci, *Risk and Asset Allocation, Exercise Book* – symmys.com