Capital Markets and Portfolio Theory

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Fall 2007, Monday 7:10 pm – 9:00 pm, Room 109

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The course covers all the aspects of quantitative portfolio management and risk management from the foundations to the most advanced developments.

- Multivariate statistics: multivariate distributions, copulas, location-dispersion ellipsoid, measures of co-dependence
- Estimation techniques: non-parametric, maximum-likelihood under thick tails, shrinkage, robust, Bayesian, extreme value theory
- Market modeling: quest for invariance in different markets, factor models, principal component analysis, FFT projection, delta-gamma & Monte Carlo pricing
- Portfolio evaluation: stochastic dominance, indices of satisfaction, utility, value at risk, expected shortfall, coherent measures
- Allocation frameworks: trading/prospect theory, total return management, benchmark allocation
- Portfolio optimization under estimation risk: Black-Litterman, Bayesian, cone programming and robust optimization

The course consists of theory and applications. The theory follows closely the adopted textbook. The applications are implemented in MATLAB® (standard, statistics and optimization toolboxes required), displayed interactively during the course to support intuition and further analyzed by the students in their homework.

Prerequisites: multivariate calculus, linear algebra, basics of probability.
No knowledge of MATLAB is assumed.

Reading: *Risk and Asset Allocation* – Springer Quantitative Finance

Grading: 40% final exam
- 60% theory: in-class, pen & paper, open-book
- 40% practice: take-home project, MATLAB

50% home assignments
10% class participation

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Office hours: Monday 5:00 pm – 7:00 pm, office 610

Lecture 1 – 09/04/07: introduction

Contents
- Announcements and course overview
- Introduction to MATLAB
    - Set path and work from command window
    - Generate scripts
    - Generate functions
    - Debug
    - Plot 2D (lines, scatter plots, histograms)
    - Plot 3D (surfaces, histograms)
- Representations of univariate distributions
    - Probability density function
    - Cumulative distribution function
    - Quantile
    - Characteristic function
- Monte Carlo simulations
    - Dirac delta and generalized functions
    - Glivenko-Cantelli theorem
    - Empirical distribution
    - Histograms and pdf
    - Empirical cdf
    - Empirical quantile by interpolation
- Distribution of transformations of random variables
    - Invertible transformations
    - Positive affine transformations
- Summary statistics:
    - Location: mode, median, expected value
    - Scale: modal dispersion, range, variance
    - Higher moments
- Taxonomy of univariate distributions
    - Uniform distribution
    - Normal/Cauchy/ Student t distributions
    - Gamma distribution
    - Lognormal distribution

References: (!) = required, (?) = optional
- (!): A. Meucci, *Risk and Asset Allocation* – Springer:
  Preface, 1.1->1.3, 4.2 p.178-179, B.1, B.2
- A. Meucci, *Risk and Asset Allocatio*n, *Technical Appendices* – symmys.com:
  (!): wherever cited above; (?): otherwise
- (!): Support materials for Lecture 1 – symmys.com

Lecture 1' – 09/07/07: MATLAB overview (by Jose Vidal Alcala)

Lecture 2 – 09/11/07: multivariate statistics I

Contents

- Representations of multivariate distributions
  - Probability density function
  - Cumulative distribution function
  - Characteristic function
  - Simulations and empirical distribution
- Copula-marginal factorization
  - Marginal distributions
  - Grades
  - Copula representation via pdf and cdf
  - Copula representation via simulations
  - Co-monotonic random variables
- Conditional distribution
  - pdf representation
  - Bayes'rule
- Dependence and concordance summary statistics
  - Special copulas
  - Schweizer-Wolff measure
  - Kendall tau
  - Spearman rho
- Simulation of generic distributions via copula and quantile

References: (!) = required, (?) = optional

- (!): A. Meucci, *Risk and Asset Allocation* – Springer:
  2.1->2.3, 2.5
- A. Meucci, *Risk and Asset Allocatio*n, *Technical Appendices* – symmys.com:
  (!): wherever cited above; (?): otherwise
- (!): Support materials for Lecture 2 – symmys.com

Lecture 3 – 09/18/07: multivariate statistics II

Contents

- Shape summary statistics
  - Affine equivariance of shape statistics
  - Expected value – covariance
  - Mode – modal dispersion
- Location-dispersion ellipsoid
  - Spectral theorem
  - Statistical interpretation
- Pearson correlation: theory, practice and pitfalls
- Taxonomy of multivariate distributions
  - Normal distribution
  - Cauchy distribution
  - Student t distribution
  - Log-distributions
  - Uniform distribution
  - Wishart distribution

References: (!) = required, (?) = optional

- (!): A. Meucci, *Risk and Asset Allocation* – Springer:
  2.4->2.6, A.1-> A.5
- A. Meucci, *Risk and Asset Allocatio*n, *Technical Appendices* – symmys.com:
  (!): wherever cited above; (?): otherwise
- (!): Support materials for Lecture 3 – symmys.com

Lecture 4 – 09/25/07: market modeling I

Contents
- Special classes of multivariate distributions
  - Order statistics
  - Elliptical distributions
  - Stable distributions
  - Infinitely divisible distributions
- The quest for invariance
  - Equities: log-returns
  - Fixed-income: changes in yield to maturity
  - Derivatives: changes in at-the-money implied volatility

References: (!) = required, (?) = optional
- (!): A. Meucci, *Risk and Asset Allocation* – Springer:
  2.6.8->2.7; 3.1
- A. Meucci, *Risk and Asset Allocatio*n, *Technical Appendices* – symmys.com:
  (!): wherever cited above; (?): otherwise
- (!): Support materials for Lecture 4 – symmys.com

Lecture 5 – 10/02/07: market modeling II

Contents
- Projection of invariants to the investment horizon
  - Convolution
  - Fourier transform
  - Analytical projection: characteristic function
  - Numerical projection: FFT
- Pricing of invariants at the investment horizon
  - Analytical: log-distributions for raw securities
  - Numerical: Monte Carlo
  - Approximate: theta-delta/vega-gamma
  - Approximate: carry-duration-convexity

References: (!) = required, (?) = optional
- (!): A. Meucci, *Risk and Asset Allocation* – Springer:
  B3, B4, 3.2, 3.3
- A. Meucci, *Risk and Asset Allocatio*n, *Technical Appendices* – symmys.com:
  (!): wherever cited above; (?): otherwise
- (!): Support materials for Lecture 5 – symmys.com

Lecture 6 – 10/09/07: market modeling III

Contents
- Dimension reduction, theory:
  - Multivariate market betas
  - Principal component analysis
- Dimension reduction, notable examples
  - Capital Asset Pricing Model
  - Arbitrage Pricing Theory
  - Fama-French factors
- Principal component analysis of the swap market
  - Level-slope-butterfly interpretation of the components
  - Continuum limit: Fourier basis and main frequencies

References: (!) = required, (?) = optional
- (!): A. Meucci, *Risk and Asset Allocation* – Springer:
  A.1->A.5, 3.4, 3.5
- A. Meucci, *Risk and Asset Allocatio*n, *Technical Appendices* – symmys.com:
  (!): wherever cited above; (?): otherwise
- (!): Support materials for Lecture 6 – symmys.com

Lecture 7 – 10/16/07: estimation I

Contents
- Estimators
  - general definitions
  - evaluation: bias, inefficiency, error
- Non-parametric estimators
  - Sample quantile and order statistics.
  - Sample mean/covariance and best-fitting ellipsoid
  - Sample factor loadings (betas) and OLS
- Maximum-likelihood estimators
  - Normal hypothesis: sample estimators
  - Non-normal hypothesis: outlier rejection

References: (!) = required, (?) = optional
- (!): A. Meucci, *Risk and Asset Allocation* – Springer:
  4.1, 4.2, 4.3
- A. Meucci, *Risk and Asset Allocatio*n, *Technical Appendices* – symmys.com:
  (!): wherever cited above; (?): otherwise
- (!): Support materials for Lecture 7 – symmys.com

Lecture 8 – 10/23/07: estimation II

Contents
- Shrinkage estimators
    - Stein mean
    - Ledoit-Wolf covariance
- Robust estimators
    - Assessing robustness: the influence function
    - Huber's "M" robust estimators: location, scatter and betas
    - Outlier detection and high-breakdown estimators
    - Minimum-volume ellipsoid and minimum-covariance determinant
- Missing data: the EM algorithm

References: (!) = required, (?) = optional
- (!): A. Meucci, *Risk and Asset Allocation* – Springer:
    4.4, 4.5, 4.6
- A. Meucci, *Risk and Asset Allocation, Technical Appendices* – symmys.com:
    (!): wherever cited above; (?): otherwise
- (!): Support materials for Lecture 8 – symmys.com

Lecture 9 – 10/30/07: estimation III

Contents
- Multivariate Bayesian estimation
    - Theoretical background
    - Analytical solutions: Normal-Inverse Wishart model
    - Numerical solutions: Monte Carlo Markov Chains

References: (!) = required, (?) = optional
- (!): A. Meucci, *Risk and Asset Allocation* – Springer:
    7.1->7.4
- A. Meucci, *Risk and Asset Allocation, Technical Appendices* – symmys.com:
    (!): wherever cited above; (?): otherwise
- (!): Support materials for Lecture 9 – symmys.com

Lecture 10 – 11/06/07: portfolio evaluation I

Contents

- Investor's objectives
  - Total return
  - Benchmark allocation
  - Net profits
- Global evaluation of a portfolio: stochastic dominance
- Summary evaluation of a portfolio: indices of satisfaction
  - Money-equivalence
  - Estimability
  - Sensibility
  - Consistence with stochastic dominance
  - Constancy
  - Positive homogeneity
  - Translation invariance
  - Sub- and super-additivity
  - Co-monotonic additivity
  - Concavity and convexity
  - Risk aversion, risk propensity and risk neutrality
- Expected utility and certainty-equivalent

References: (!) = required, (?) = optional

- (!): A. Meucci, *Risk and Asset Allocation* – Springer:
  5.1->5.4
- A. Meucci, *Risk and Asset Allocatio*n, *Technical Appendices* – symmys.com:
  (!): wherever cited above; (?): otherwise
- (!): Support materials for Lecture 10 – symmys.com

Lecture 11 – 11/13/07: portfolio evaluation II

Contents
- Quantiles and value at risk (VaR)
  - Properties
  - Analytical solutions
  - Cornish-Fisher approximation
  - Extreme value theory (EVT)
  - Numerical solutions
- Coherent measures of performance
  - Expected shortfall (ES) and conditional value at risk (CVaR)
  - Spectral measures of performance

References: (!) = required, (?) = optional
- (!): A. Meucci, *Risk and Asset Allocation* – Springer:
  5.5->5.6
- A. Meucci, *Risk and Asset Allocatio*n, *Technical Appendices* – symmys.com:
  (!): wherever cited above; (?): otherwise
- (!): Support materials for Lecture 11 – symmys.com

Lecture 12 – 11/20/07: portfolio optimization I

Contents
- Portfolio optimization theory
  - Investor's inputs: market, investment horizon, objectives and satisfaction
  - Market inputs: distribution of prices at the horizon, transaction costs
- Constrained optimization: computationally tractable problems
  - Linear and quadratic programming
  - Second order and semi-definite cone programming
- Two-step optimization
  - Analytical solutions
  - Numerical solutions
- Benchmark vs. total-return portfolio management
  - Mean-variance approximation
  - Analytical solutions in total-return coordinates
  - Analytical solutions in relative-return coordinates:
    expected outperformance, tracking error, information ratio
  - Pitfalls of the mean-variance approach

References: (!) = required, (?) = optional
- A. Meucci, *Risk and Asset Allocation* – Springer:
  (!): 6.1->6.7
- A. Meucci, *Risk and Asset Allocatio*n, *Technical Appendices* – symmys.com:
  (!): wherever cited above; (?): otherwise
- (!): Support materials for Lecture 12 – symmys.com

Lecture 13 – 11/27/07: portfolio optimization II

Contents
- Prior allocation
- Sample-based allocation
    - Error in satisfaction and constraint assessment
    - Leverage of estimation risk
- Alternative optimization methods
- Allocations as decisions
    - Opportunity cost
    - Allocation decisions evaluated as estimators

References: (!) = required, (?) = optional
- (!): A. Meucci, *Risk and Asset Allocation* – Springer:
    8.1 -> 8.3
- A. Meucci, *Risk and Asset Allocatio*n, *Technical Appendices* – symmys.com:
    (!): wherever cited above; (?): otherwise
- (!): Support materials for Lecture 13 – symmys.com

Lecture 14 – 12/04/07: portfolio optimization III

Contents
- Bayesian allocation
    - Predictive return allocation
    - Classical-equivalent allocation
- Black-Litterman allocation
    - Views on market parameters
    - Views on the market realizations
- Copula-opinion pooling allocation
- Resampled allocation
- Robust allocation
    - Second-order cone programming problems
    - Semi-definite programming problems
- Robust Bayesian allocation

References: (!) = required, (?) = optional
- (!): A. Meucci, *Risk and Asset Allocation* – Springer:
    9.1 -> 9.5
- A. Meucci, *Risk and Asset Allocatio*n, *Technical Appendices* – symmys.com:
    (!): wherever cited above; (?): otherwise
- (!): Support material for Lecture 14 – symmys.com