Attilio Meucci
Lehman Brothers, Inc., New York

Quantitative Portfolio- and Risk-Management

PhD Program in Economics
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The course covers quantitative portfolio management from the foundations to the most advanced developments.
The discussion follows closely the textbook *Risk and Asset Allocation - Springer (2005)*. The theory is supported by practical examples that are implemented in MATLAB®. The software will be made available to the students attending the course.

Synopsis
Multivariate estimation methods are analyzed, including non-parametric, maximum-likelihood under non-normal hypotheses, shrinkage, robust, and general Bayesian techniques. Portfolio evaluation methods such as stochastic dominance, expected utility, value at risk and coherent measures are discussed in a unified setting and applied in a variety of contexts, including prospect theory, total return and benchmark allocation.
Classical portfolio optimization is discussed in a general setting and feasible approaches such as mean-variance and mean-CVaR are analyzed. Optimization under estimation risk is then thoroughly discussed: the Black-Litterman approach, more general Bayesian approaches, the resampling procedure and robust optimization techniques, which can be solved by means of cone-programming.

Prerequisites: probability, multivariate calculus and linear algebra. Appendix A and B in the textbook are required reading before the course begins.

Course webpage: symmys.com > Teaching

Lecture 1 – 03/13/06: multivariate statistics

- Review of univariate statistics
  - Building blocks: pdf, cdf, quantiles, characteristic function
  - Summary statistics
  - Taxonomy of univariate distributions
- Glivenko-Cantelli theorem and Monte Carlo simulations
- Copulas vs. marginal factorization of a joint distribution
- Dependence summary statistics
  - Kendall tau
  - Spearman rho
  - Pearson correlation
- Shape summary statistics
  - Expected value – covariance
  - Mode – modal dispersion
  - Location-dispersion ellipsoid
- Taxonomy of distributions
  - Elliptical uniform distribution
  - Matrix-variate normal distribution
  - Matrix-variate t distribution
  - Log-distributions
  - Wishart distribution
  - Empirical distribution
  - Order statistics

Lecture 2 – 03/14/06: estimation techniques

- Estimators
  - general definitions
  - evaluation: bias, inefficiency, error
- Non-parametric estimators
  - Sample quantile and order statistics.
  - Sample mean/covariance and best-fitting ellipsoid
  - Sample factor loadings and ordinary least squares
  - Kernel estimators
- Maximum-likelihood estimators
  - Normal hypothesis: sample estimators
  - Non-normal hypothesis: outlier rejection
- Shrinkage estimators
  - Stein mean
  - Ledoit-Wolf covariance
- Robust estimators
  - Assessing robustness: the influence function
  - Huber's "M" robust estimators
  - Outlier detection and high-breakdown estimators
  - Minimum-volume ellipsoid and minimum-covariance determinant
- Bayesian estimators
  - Analytically tractable examples
  - Numerical techniques
- Missing observations: estimation from unbalanced panels
  - E-M algorithm
  - ML marginalization

Lecture 3 – 03/15/06: market modeling and portfolio evaluation

- The quest for invariance
  - Equities: log-returns
  - Fixed-income: changes in yield to maturity
  - Derivatives: changes in at-the-money implied volatility
- Projection of invariants to the investment horizon
  - Analytical: characteristic function
  - Numerical: FFT
- Pricing of invariants at the investment horizon
  - Analytical: log-distributions for raw securities
  - Numerical: Monte Carlo
  - Approximate: theta-delta/vega-gamma
- Investor's objectives
  - Total return
  - Benchmark allocation
  - Net profits
- Global evaluation of a portfolio: stochastic dominance
- Summary evaluation of a portfolio: indices of satisfaction
  - Money-equivalence
  - Estimability
  - Sensibility
  - Consistence with stochastic dominance
  - Constancy
  - Positive homogeneity
  - Translation invariance
  - Sub- and super-additivity
  - Co-monotonic additivity
  - Concavity and convexity
  - Risk aversion, risk propensity and risk neutrality
- Expected utility and certainty equivalent
- Quantiles and value at risk (VaR)
- Coherent measures of performance
  - Expected shortfall (ES) and conditional value at risk (CVaR)
  - Spectral measures of performance

Lecture 4 – 03/16/06: classical portfolio optimization

- The general framework: a guided tour
- Constrained optimization: computationally tractable problems
  - Linear and quadratic programming
  - Second order and semi-definite cone programming
- Mean-variance optimization
  - Analytical solutions: total return vs. benchmark allocation
  - Numerical solutions: quadratic programming
  - Pitfalls of the mean-variance approach
- Market asymmetries and the Mean-CVaR approach
  - Theoretical framework
  - Numerical solutions: linear programming

Lecture 5 – 03/17/06: portfolio optimization with estimation risk

- Allocations as decisions
  - Opportunity cost
  - Allocation decisions evaluated as estimators
- Prior allocation
- Sample-based allocation: using naïve estimates in portfolio management
  - Error in satisfaction and constraint assessment
  - Leverage of estimation risk
- Bayesian allocation
  - Predictive return allocation
  - Classical-equivalent allocation
- Black-Litterman
  - Views on market parameters
  - Views on the market realizations
- Copula-Opinion Pooling allocation
- Resampled allocation
- Robust allocation
  - Second-order cone programming problems
  - Semi-definite programming problems
- Robust Bayesian allocation